Journal of Mathematics Education
December 2009, Vol. 2, No. 2, pp.147-164

Characteristics of Effective Teaching of Mathematics: A View from the West

Glenda Anthony and Margaret Walshaw
Massey University, New Zealand

Worldwide, policy makers are placing increasing demands on schools and their teachers to use effective research-informed practices. In New Zealand a collaborative knowledge building strategy—The Iterative Best Evidence Synthesis Program—has been implemented at policy level. Drawing on findings from the mathematics Best Evidence Synthesis Iteration, and more recent research studies, this paper offers ten principles of effective pedagogical approaches that facilitate learning for diverse learners. In examining the links between pedagogical practices and a range of social and academic student outcomes we draw on the histories, cultures, language, and practices for the New Zealand context and comparable international contexts.

Key words: mathematics pedagogy, effective teaching, community of learners, tasks, discourse, teacher knowledge.

Mathematics, it is widely understood, plays a key role in shaping how individuals deal with the various spheres of private, social, and civil life. Yet today, as in the past, many students struggle with mathematics and become disaffected as they continually confront obstacles to engagement. In order to break this pattern it is imperative, therefore, that we understand what effective mathematics teaching looks like. Many have looked to research to seek evidence about what kinds of pedagogical practices contribute to desirable student outcomes (see government funded reports by, for example, Anthony & Walshaw, 2007; Doig, McCrae, & Rowe, 2003; Ingvarson, Beavis, Bishop, Peck, & Ellsworth, 2004; National Mathematics Advisory Panel, 2008). Hiebert and Grouws (2007), in their synthesis of international research, have argued for a more detailed, richer, and coherent knowledge base to inform policy and practice.

In a response to Hiebert and Grouws, we present findings from recent research syntheses (Anthony & Walshaw, 2007; 2008), complemented by evidence from recent international studies (e.g., Lester, 2007; Martin, 2007).

Collectively, these reviews are closely aligned with recent mathematics initiatives within western education systems that shift teaching and learning away from a traditional emphasis on learning rules for manipulating symbols. Initiatives like Principles and Standards for School Mathematics (PSSM) (National Council of Teachers of Mathematics, 2000) focus on developing communities of practice in which students are actively engaged with mathematics.

Effective pedagogy within such communities is at the heart of this paper. We ask: What does research tell us about the characteristics of effective pedagogy in the west? From our investigations that have helped us answer that question, we have developed a set of principles that underpin the kinds of pedagogical approaches found to develop mathematical capability and disposition within an effective learning community. The ten principles of effective mathematics pedagogy should not be taken in isolation but interpreted as part of a complex web of factors that can affect student learning. They incorporate elements of practice related to the classroom community, classroom discourse, the kinds of tasks that enhance students' thinking, and the role of teacher knowledge (see Figure 1). We discuss each of these principles, in turn, in the following sections.

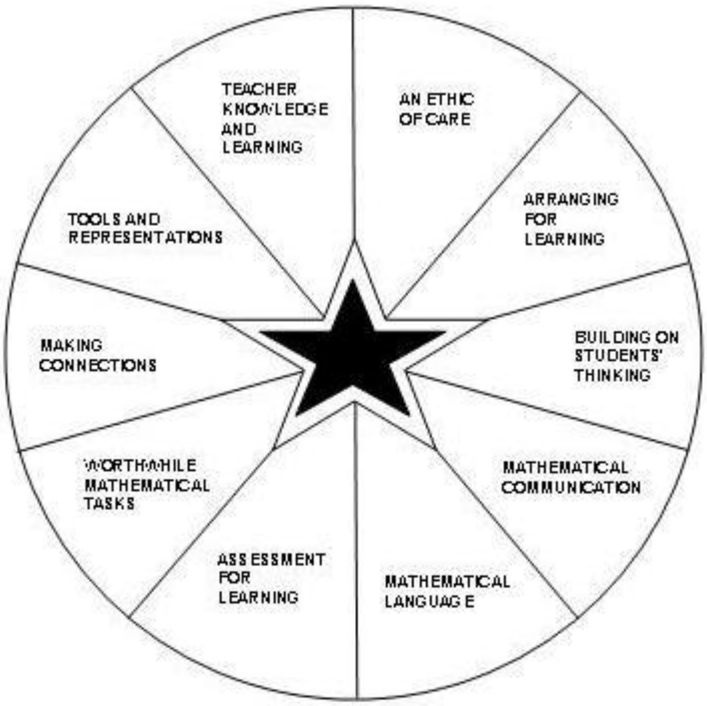

Figure 1. **Principles of effective pedagogy of mathematics**

The principles we have developed are based on recognition that classroom teaching is a complex activity. The classroom learning community is neither static nor linear. Rather, it is nested within an evolving network involving the school, the wider education system, and the home and local community. The idea that teaching sits within a nested system draws its inspiration from the work of post-Vygotskian activity theorists such as Davydov and Radzikhovskii (1985). The understanding of a close relationship between social processes and conceptual development also forms the basis of Lave and Wenger's (1991) well-known social practice theory, in which the notions of "a community of practice" and "the connectedness of knowing" are central features. In that theoretical framework, individual and collective knowledge emerge and evolve within the dynamics of the spaces people share and within which they participate.

In this paper our focus will be on the classroom as a community of practice. Our starting point is in the understanding that teachers who foster positive student outcomes do so through their beliefs in the rights of all students to have access to mathematics education in a broad sense—understanding of the big ideas of curriculum and an appreciation of their value and application in everyday life. Additionally, we claim that effective mathematics pedagogy:

1) acknowledges that all students, irrespective of age, can develop positive mathematical identities and become powerful mathematical learners.

2) is based on interpersonal respect and sensitivity and is responsive to the multiplicity of cultural heritages, thinking processes, and realities found in everyday classrooms.

3) is focused on optimizing a range of desirable academic outcomes that include conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning.

4) is committed to enhancing a range of social outcomes within the mathematics classroom that will contribute to the holistic development of students for productive citizenship.

Classroom Community

An Ethic of Care: Caring Classroom Communities that Are Focused on Mathematics Goals Help Develop Students' Mathematical Identities and Proficiencies

From research studies we find that effective teachers facilitate learning by truly caring about their students' engagement (Noddings, 1995). They work at developing interrelationships that create spaces for students to develop their mathematical and cultural identities. They have high yet realistic expectations about enhancing students' capacity to think, reason, communicate, reflect upon and critique their own practice, and they provide students opportunities to ask why the class is doing certain things and with what effect (Watson, 2002). The relationships that develop in the classroom become a resource for developing students' mathematical competencies and identities.

Students want to learn in a 'togetherness' environment (Boaler, 2008; Ingram, 2008). Teachers can make everyone feel included by respecting and valuing the mathematics and the cultures that students bring to the classrooms. Ensuring that all students feel safe allows every student to get involved. However, it is important that the caring relationships that are developed do not encourage students to become overly dependent on their teachers. Effective teachers promote classroom relationships that allow students to think for themselves, to ask questions, and to take intellectual risks (Angier & Povey, 1999).

Everyday classroom routines play an important role in the development of students' mathematical thinking. Effective teachers make sure that all students are provided with opportunities to struggle with mathematics for themselves. For example, simply inviting students to contribute a response to a mathematical problem may not achieve anything more than cooperation from students. Teachers need to provide students with expectations and obligations concerning who might speak, when and in what form, and what listeners might do (Stipek et al., 1998).

Teachers are the most important resource for developing students' mathematical identities (Cobb & Hodge, 2002). They influence the ways in which student's think of themselves in the classroom (Walshaw, 2004). In establishing equitable arrangements, effective teachers pay attention to the different needs that result from different home environments, different languages, and different capabilities and perspectives. The positive attitude that develops raises students' comfort level, enlarges their knowledge base, and gives them greater confidence in their capacity to learn and make sense of mathematics. Confident in their own understandings, students will be more willing to consider new ideas presented by the teacher, to consider other students' ideas and assess the validity of other approaches, and to persevere in the face of mathematical challenge.

Arranging for Learning: Effective Teachers Provide Students with Opportunities to Make Sense of Ideas both Independently and Collaboratively

An important role of the teacher is to provide students with working arrangements that are responsive to their needs. All students need some time to think and work quietly by themselves, away from the varied and sometimes conflicting perspectives of other students (Sfard & Keiran, 2001). At other times, partners or peers in groups can provide the context for sharing ideas and for learning with and from others. Group or partner arrangements are useful not only for enhancing engagement but also for exchanging and testing ideas and generating a higher level of thinking (Ding, Li, Piccolo, & Kulm, 2007). In supportive, small-group environments, students learn to make conjectures and learn how to engage in mathematical argumentation and validation (O'Conner & Michaels, 1996). In particular, when groups are mixed in relation to academic achievement, insights are provided at varying levels within the group, and these insights tend to enhance overall understandings However, teachers need to clarify expectations of participation and ensure that roles for participants, such as listening, writing, answering, questioning, and critically assessing, are understood and implemented (Hunter, 2008).

Whole class discussion can provide a forum for broader interpretations and an opportunity for students to clarify their understanding. It can also assist students in solving challenging problems when a solution is not initially available. Teachers have an important role to play in the discussion. Focusing attention on efficient ways of recording, they invite students to listen to and respect one another's solutions and evaluate different viewpoints. In all forms of classroom organization it is the teacher's task to listen, to monitor how often students contribute, and to keep the discussion focused. When class discussion is an integral part of an overall strategy for teaching and learning, students provide their teachers with information about what they know and what they need to learn.

Building on students' thinking: Effective teachers plan mathematics learning experiences that allow students to build on their existing proficiencies, interest, and experiences.

In planning for learning, effective teachers put students' current knowledge and interests at the centre of their instructional decision making. Informed by on-going assessment of students' competencies, including language, reading and listening skills, ability to cope with complexity, and mathematical reasoning, teachers adjust their instruction to meet the learning needs of their students.

With the emphasis on building on students' existing proficiencies, rather than remediating weaknesses and filling gaps in students' knowledge, effective teachers are able to be both responsive to their students and to the discipline (Carpenter, Fennema, & Franke, 1996). They understand that learners make mistakes for many reasons. Some mistakes happen because students have not taken sufficient time or care; others are the result of consistent, alternative interpretations of mathematical ideas that arise from learners' attempts to create meaning. To help students to learn from their errors, teachers organize discussions—with peers or the whole class—that focus students' attention on the known difficulties. Asking students to share a variety of interpretations or solution strategies enables learners to compare and re-evaluate their ideas.

Teachers who start where students are at with their learning are also able to design appropriate levels of challenges for their students. For low-achieving students, teachers find ways to reduce the complexity of tasks without falling back on repetition and busywork and without compromising the mathematical integrity of the activity (Houssart, 2002). In order to increase the task challenge in all classrooms, effective teachers put obstacles in the way of solutions, remove some information, require the use of particular representations, or ask for generalizations (Sullivan, Mousley, & Zevenbergen, 2006).

Discourse in the Classroom

Mathematical Communication: Effective Teachers Facilitate Classroom Dialogue that Is Focused Towards Mathematical Argumentation

Teaching ways of communicating mathematically demands skilful work on the teacher's part (Walshaw & Anthony, 2008). Students need to be taught how to articulate sound mathematical explanations and how to justify their solutions. Encouraging the use of oral, written and concrete representations, effective teachers model the process of explaining and justifying, guiding students into mathematical conventions. They use explicit strategies, such as telling students how they are expected to communicate (Hunter, 2005).

Teachers can also use the technique of revoicing (Forman & Ansell, 2001), repeating, rephrasing, or expanding on student talk. Teachers use revoicing in many ways: (i) to highlight ideas that have come directly from students, (ii) to help the development of students' understandings implicit in those ideas, (iii) to negotiate meaning with their students, and (iv) to add new ideas, or move discussion in another direction.

When guiding students into ways of mathematical argumentation, it is important that the classroom learning community allows for disagreements and enables conflicts to be resolved (Chapin & O'Connor, 2007). Teachers' support should involve prompts for students to work more effectively together, to give reasons for their views and to offer their ideas and opinions. Students and teacher both need to listen to others' ideas and to use debate to establish common understandings. Listening attentively to student ideas helps teachers to determine when to step in and out of the discussion, when to press for understanding, when to resolve competing student claims, and when to address misunderstandings or confusion (Lobato, Clarke, & Ellis, 2005). As students' attention shifts from procedural rules to making sense of mathematics, students become less preoccupied with finding the answers and more with the thinking that leads to the answers (Fravillig, Murphy, & Fuson, 1999).

Mathematical Language: The Use of Mathematical Language Is Shaped When the Teacher Models Appropriate Terms and Communicates Their Meaning in a Way that Students Understand

If students are to make sense of mathematical ideas they need an understanding of the mathematical language used in the classroom. A key task for the teacher is to foster the use, as well as the understanding, of appropriate mathematical terms and expressions. Conventional mathematical language needs to be modeled and used so that, over time, it can migrate from the teacher to the students (Runesson, 2005). Explicit language instruction and modeling takes into account students' informal understandings of the mathematical language in use. For example, words such as "less than", "more", "maybe", and "half" can have quite different meanings within a family setting. Students can also be helped in grasping the underlying meaning through the use of words or symbols with the same mathematical meaning, for example, 'x', 'multiply', and 'times'.

Teachers face particular challenges in multilingual classrooms. Words such as "absolute value", "standard deviation", and "very likely" often lack an equivalent term in the students' home language. Students find the syntax of mathematical discourse difficult. Prepositions, word order, logical structures, and conditionals are all particularly problematic for students. Students may also be unfamiliar with the contexts in which problems have been situated. Language (or code) switching, which involves the teacher substituting a home language word for a mathematical word, has been shown to enhance student understanding, especially when teachers are able to use it to capture the specific nuances of mathematical language (Setati & Adler, 2001).

Assessment for learning: Effective teachers use a range of assessment practices to make students' thinking visible and support students' learning.

Mathematics teachers make use of a wide range of formal and informal assessments to monitor learning progress, to diagnose learning, and to determine what can be done to improve learning. Within the everyday activities of the classroom, teachers collect information about how students learn, what they seem to know and are able to do, and what they are interested in. This information helps teachers determine whether particular activities are successful and informs decisions about what they should be doing to meet the learning needs of their students (Wiliam, 2007).

Effective teachers gather information about students by watching students as they engage in individual or group work and by talking with them. They monitor their students' understanding, notice the strategies that they prefer, and listen to the language that they use. The moment-by-moment assessment helps them make decisions about what questions to ask next, when to intervene in student activity, and how to answer questions. Classroom exchanges in the form of careful questioning provide a powerful way to assess students' current knowledge and ways of thinking (Steinberg, Empson, & Carpenter, 2004). For example, questions that have a variety of solutions, or that can be solved in more than one way, can help teachers gain insight into students' mathematical thinking and reasoning.

As well as informing the teacher, assessment for learning involves providing feedback to students. Helpful feedback explains why something is right or wrong, and describes what to do next, or describes strategies for improvement. Effective teachers also provide opportunities for their students to evaluate and assess their own work. They involve students in designing test questions, writing success criteria, writing mathematical journals, and presenting portfolios as evidence of growth in mathematics.

Mathematical Tasks

Worthwhile Tasks: Effective Teachers Understand that Selected Tasks and Examples Influence How Students Come to View, Develop, Use, and Make Sense of Mathematics

Tasks convey what doing mathematics is all about. By engaging in tasks, students develop ideas about the nature of mathematics and mathematics learning (Hodge, Zhao, Visnovska, & Cobb, 2007). Effective teachers take care to ensure that tasks help all students to progress in their cumulative understanding in a particular domain and engage in high-level mathematical thinking (Henningsen & Stein, 1997).

By posing tasks and learning experiences that allow students to do original thinking about important mathematical concepts and relationships, teachers help learners to develop proficient ways of doing, and learning about mathematics (Ainley, Pratt, & Hansen, 2006). Tasks should involve more than practicing taught algorithms; they should provide opportunities for students to struggle with important mathematical ideas. Posing tasks of an appropriate level of mathematical challenge fosters students' development and use of an increasingly sophisticated range of mathematical thinking and reasoning activities (Watson & De Geest, 2005).

Working with open-ended and modeling tasks, in particular, provides students with opportunities not just to apply mathematics but also to learn new mathematics through engagement in a range of problem-solving strategies. Essential skill development can also be part of 'doing' mathematics problems. For example, learning about perimeter and area provides opportunities to practice multiplication and fractions computations. Modeling activities challenge students to make sense of both the contexts and the mathematics embedded in the tasks (English, 2006; Galbraith, Stilman, Brown, & Edwards, 2007). When working with real life complex systems, students learn that doing mathematics involves more than simply producing right answers; applying mathematics in everyday settings helps students learn about the value of mathematics in society and its contributions to other disciplines.

Making Connections: Effective Teachers Support Students to Create Connections, between Different Ways of Solving Problems, between Mathematical Topics, and between Mathematics and Everyday Experiences

Students need to develop understandings of how a concept or skill is connected in multiple ways to other mathematical ideas (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997). Effective teachers support students to make connections by providing them with opportunities to engage in complex tasks and by setting expectations that they explain their thinking and solution strategies and that they listen to the thinking of others (Anghileri, 2006). Teachers can assist students to make connections by using carefully sequenced examples, including examples of students' own solution strategies, to illustrate key mathematical ideas (Watson & Mason, 2006). By progressively introducing modifications that build on students' existing understanding, teachers can emphasize the links between different ideas in mathematics. For example, a teacher can introduce the idea of 'doubling 6' as an alternative strategy to '6 add 6'.

Making connections across mathematical topics is important for developing conceptual understanding. For example, the topics of fractions, decimals, percentages, and proportions, while learning areas in their own right, can usefully be linked through exploration of differing representations (e.g., ½ = 50%) or through problems involving everyday contexts (e.g., determining fuel costs for a car trip).

Teachers can also help students to make connections to real experiences. When students find they can use mathematics as a tool for solving significant problems in their everyday lives, they begin to view the subject as relevant and interesting.

Tools and Representations: Effective Teachers Carefully Select Tools and Representations to Provide Support for Students' Thinking.

Effective teachers draw on a range of representations and tools to support learners' mathematical development. Tools to support and extend mathematical reasoning and sense-making come in many forms including the number system itself, algebraic symbolism, graphs, diagrams, models, equations, notations, images, analogies, metaphors, stories, textbooks, and technology.

Teachers have a critical role to play in ensuring that tools are used effectively to support students to organize their mathematical reasoning and support their sense-making (Blanton & Kaput, 2005). Providing students access to multiple representations helps them to develop conceptual and computational flexibility. Using an appropriate model, learners can think through a problem, or test ideas. Care is needed, however, particularly when using pre-designed concrete materials (e.g., number lines, tens-frames), to ensure that all students are able to make sense of the materials in the mathematically intended way.

Tools are helpful in communicating ideas that are otherwise difficult to talk about or write about. Teachers and students can use representations, such as stories, pictures, symbols, concrete objects, and virtual manipulatives, to assist in communicating their thinking to others. As well as ready-made tools, effective teachers acknowledge the value of students generating and using their own representations, but it an invented notation, or a graphical, pictorial, tabular, or geometric representation. For example, young children frequently create their own pictorial representations to tell stories before using the more formal graphical tools that are fundamental to the statistics curriculum (Chick, Pfannkuch, & Watson, 2005).

An increasing array of new technological tools is available for use in the mathematics classroom. In addition to calculator and computer applications, new technologies include presentation technologies (e.g., the interactive whiteboard (Zevenbergen & Lerman, 2008), digital and mobile technologies, and the Internet. These dynamic graphical, numerical, and visual technological applications provide new opportunities for teachers and students to interact, represent, and explore mathematical concepts.

Teachers must be knowledgeable decision makers in determining when and how technology is used to support learning (Thomas & Chinnappan, 2008). Effective teachers take time to share the decision making about technology-based approaches with their students. They require students to monitor their own underuse or overuse of technology. With guidance from teachers, technology can support independent inquiry and shared knowledge building.

Teacher Learning and Knowledge

Teacher knowledge: Effective Teachers Develop and Use Sound Knowledge to Initiate Learning and to Act Responsively towards the Mathematical Needs of All Their Students

How teachers organize classroom instruction is very much dependent on what they know and believe about mathematics and on what they understand about mathematics teaching and learning. Sound content knowledge enables teachers to represent mathematics as a coherent and connected system (Ball & Bass, 2000). When their knowledge is robust, teachers are able to assess their students' current level of mathematical understanding. They use their knowledge to make key decisions concerning mathematical tasks, classroom resources, talk, and actions that feed into or arise out of the learning process.

No matter how good their teaching intentions, teachers must work out how they can best help their students grasp core mathematical ideas (Hill, Rowan, & Bass, 2005). In addition to having clear ideas about how they might build students' procedural proficiency they need to know how to extend and challenge students' thinking. To do this successfully they need substantial pedagogical content knowledge and a grounded understanding of students as learners. Such teachers are aware of the possibility of students' conceptions and misconceptions. This knowledge informs teachers' on-the-spot classroom decision making. It enables more finely tuned listening and questioning, more focused and connected planning, and more insightful evaluation of student responses.

The development of teacher knowledge is greatly enhanced by efforts within the wider school community to improve teachers' own understandings of mathematics and mathematics teaching and learning (Cobb & McClain, 2001; Sherin, 2002). If teachers' knowledge is to be enhanced, it needs the material, systems, human and emotional support provided by professional development initiatives. Support and resourcing can also come from the joint efforts of other mathematics teachers within the school (Kazemi, 2008).

This paper has examined what the research says about effective teaching of mathematics within western education systems. Current research findings indicate that the nature of classroom mathematics teaching significantly affects the nature and outcome of student learning. Our conceptualization of teaching as nested within a systems network (Tower & Davis, 2002), moves us away from prescribing pedagogical practice, towards an understanding of pedagogical practice as occasioning students outcomes. In this paper we have offered important insights from research about how that occasioning might take place. Certain patterns have emerged that have enabled us to foreground ways of doing and being that mark out an effective pedagogical practice. Each aspect, of course, constitutes but one piece of evidence and must be read as accounting for one variable, amongst many, within the teaching nested system. As Hiebert and Grouws (2007) have noted, "classrooms are filled with complex dynamics, and many factors could be responsible for increased student learning" (p. 371). Taking all the factors together has allowed us to offer our ten principles as a starting point for discussions on effective pedagogy.

Whilst the principles concern classroom pedagogical practices, we are well aware that significant improvements in student learning outcomes will require the efforts of many. Changes need to be negotiated and carried through in classrooms; in mathematics teams, departments, or faculties; and in teacher education programs. They need to be supported by resourcing. Everyone involved in mathematics education—teachers, principals, teacher educators, researchers, parents, specialist support services, school boards, and policy makers, as well as students themselves—has a role to play in enhancing students' mathematical proficiency. Schools, communities, and nations need to ensure that their teachers have the knowledge, skills, resourcing, and incentives to provide students with the very best possible learning opportunities. In this way, every student will be able to enhance their mathematical proficiency. In this way, too, every student has the opportunity to enhance their view of themselves as a powerful mathematics learner.

References

Ainley, J., Pratt, D., & Hansen, A. (2006). Connecting engagement and focus in pedagogic task design. *British Educational Research Journal, 32*(1), 23–38.

Anghileri, J. (2006). Scaffolding practices that enhance mathematics learning. *Journal of Mathematics Teacher Education, 9*, 33-52.

Angier, C., & Povey, H. (1999). One teacher and a class of school students: Their perception of the culture of their mathematics classroom and its construction. *Educational Review, 51*, 147-160.

Anthony, G., & Walshaw, M. (2007). *Effective pedagogy in mathematics/pāngarau: Best evidence synthesis iteration [BES]*. Wellington: Ministry of Education.

Anthony, G., & Walshaw, M. (2008). Characteristics of effective pedagogy for mathematics education. In H. Forgasz, T. Barkatsas, A. Bishop, B. Clarke, P. Sullivan, S. Keast, W. T. Seah, & S. Willis (Eds.), *Research in mathematics education in Australasia 2004-2007* (pp. 195-222). Rotterdam Netherlands: Sense.

Askew, M., Brown, M., Rhodes, V., Johnson, D., & Wiliam, D. (1997). *Effective teachers of numeracy*. London: Kings College.

Ball, D., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83–104). Westport, CT: Ablex.

Blanton, M., & Kaput, J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *Journal for Research in Mathematics Education, 36*, 412-446.

Boaler, J. (2008). Promoting 'relational equity' and high mathematics achievement through an innovative mixed-ability approach. *British Educational Research Journal, 34*, 167-194.

Carpenter, T., Fennema, E., & Franke, M. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal, 97*(1), 3-20.

Chapin, S. H., & O'Connor, C. (2007). Academically productive talk: Supporting students' learning in mathematics. In W. G. Martin, M. Strutchens, & P. Elliot (Eds.), *The learning of mathematics* (pp. 113-139). Reston VA: NCTM.

Chick, H., Pfannkuch, M., & Watson, J. (2005). Transnumerative thinking: Finding and telling stories within data. *Curriculum Matters, 1*, 86-107.

Cobb, P., & Hodge, L. L. (2002). A relational perspective on issues of cultural diversity and equity as they play out in the mathematics classroom. *Mathematical Thinking and Learning, 4*, 249–284.

Cobb, P., & McClain, K. (2001). An approach for supporting teachers'' learning in social context. In F. Lin & T. Cooney (Eds.), *Making sense of mathematics teacher education* (pp. 207-231). Utrecht: Kluwer Academic Publishers.

Davydov, V.V., & Radzikhovskii, L.A. (1985). Vygotsky's theory and the activity-oriented approach in psychology. In J.V. Wertsch (Ed.), *Culture, communication, and cognition: Vygotskian perspectives* (pp. 35-65). New York: Cambridge University Press.

Ding, M., Li, X., Piccolo, D., & Kulm, G. (2007). Teaching interventions in cooperative learning mathematics classes. *The Journal of Educational Research, 100*, 162-175.

Doig, B., McCrae, B., & Rowe, K. J. (2003). *A good start to numeracy: Effective numeracy strategies from research and practice in early childhood*: Australian Council of Educational Research.

English, L. D. (2006). Mathematical modeling in the primary school: Children's construction of a consumer guide. *Educational Studies in*

Mathematics, 63, 303–323.

Forman, E., & Ansell, E. (2001). The multiple voices of a mathematics classroom community. *Educational Studies in Mathematics, 46*, 114–142.

Fraivillig, J., Murphy, L., & Fuson, K. (1999). Advancing children's mathematical thinking in Everyday Mathematics classrooms. *Journal for Research in Mathematics Education, 30*, 148–170.

Galbraith, P., Stillman, G., Brown, J., & Edwards, I. (2007). Facilitating middle secondary modelling competencies. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modeling: Education, engineering and economic* (pp. 130–140). Chichester, UK: Horwood.

Henningsen, M., & Stein, M. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, *28*, 524–549.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371-404). Charlotte, NC: Information Age Publishers.

Hill, H., Rowan, B., & Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Education Research Journal*, *42*, 371-406.

Hodge, L., Zhao, Q., Visnovska, J., & Cobb, P. (2007). What does it mean for an instructional task to be effective? In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice* (Vol. 1, pp. 329–401). Hobart: MERGA.

Houssart, J. (2002). Simplification and repetition of mathematical tasks: A recipe for success or failure? *The Journal of Mathematical Behavior, 21(2)*, 191–202.

Hunter, R. (2005). Reforming communication in the classroom: One teacher's journey of change. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building connections: Theory, research and practice* (Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia, Vol. 1, pp. 451-458). Melbourne: MERGA.

Hunter, R. (2008). Facilitating communities of mathematical inquiry. In M. Goos, R. Brown, & R. Makar (Eds.), *Navigation currents and charting directions* (Proceedings of the 31st annual Mathematics Education

Research Group of Australasia conference, Vol. 1, pp. 31-39). Brisbane Australia: MERGA.

Ingram, N. (2008). Who a student sits near to in maths: Tension between social and mathematical identities. In M. Goos, R. Brown, & R. Makar (Eds.), *Navigation currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, Vol. 1, pp. 281-286). Brisbane Australia: MERGA.

Ingvarson, L., Beavis, A., Bishop, A., Peck, R., & Elsworth, G. (2004). *Investigation of effective mathematics teaching and learning in Australian secondary schools*. Canberra, Australian: Council for Educational Research.

Kazemi, E. (2008). School development as a means of improving mathematics teaching and learning. In K. Krainer & T. Wood (Eds.), *Participants in mathematics teacher education* (pp. 209-230). Rotterdam Netherlands: Sense.

Lester, F. (Ed.). (2007). *Second handbook of research on mathematics teaching and learning* (Vol. 1 & 2). Reston VA: NCTM.

Lobato, J., Clarke, D., & Ellis, A. B. (2005). Initiating and eliciting in teaching: A reformulation of telling. *Journal for Research in Mathematics Education, 36*(2), 101–136.

Martin, T. S. (Ed.) (2007) *Mathematics teaching today: Improving practice, improving student learning* (2nd ed.). Reston, VA: Author.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston VA: Author.

National Research Council (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

Noddings, N. (1995). *Philosophy of education*. Oxford: Westview Press.

O'Connor, M. C., & Michaels, S. (1996). Shifting participant frameworks: Orchestrating thinking practices in group discussion. In D. Hicks (Eds.), *Discourse, learning and schooling* (pp. 63–103). New York: Cambridge University Press.

Runesson, U. (2005). Beyond discourse and interaction. Variation: a critical aspect for teaching and learning mathematics. *Cambridge Journal of Education, 35*(1), 69-87.

Setati, M., & Adler, J. (2001). Code-switching in a senior primary class of secondary-language mathematics learners. *For the Learning of Mathematics, 18,* 34–42.

Sfard, A., & Keiran, C. (2001). Cognition as communication: Rethinking learning-by-talking through multi-faceted analysis of students' mathematical interactions. *Mind, Culture, and Activity, 8*(1), 42–76.

Sherin, M. G. (2002). When teaching becomes learning. *Cognition and instruction, 20*(2), 119–150.

Steinberg, R. M., Empson, S. B., & Carpenter, T. P. (2004). Inquiry into children's mathematical thinking as a means to teacher change. *Journal of Mathematics Teacher Education, 7,* 237–267.

Stipek, D., Salmon, J. M., Givvin, K. B., Kazemi, E., Saxe, G., & MacGyvers, V. L. (1998). The value (and convergence) of practices suggested by motivation research and promoted by mathematics education reformers. *Journal for Research in Mathematics Education, 29,* 465–488.

Sullivan, P., Mousley, J., & Zevenbergen, R. (2006). Teacher actions to maximize mathematics learning opportunities in heterogeneous classrooms. *International Journal of Science and Mathematics Education, 4*(1), 117–143.

Thomas, M., & Chinnappan, M. (2008). Teaching and learning with technology: Realizing the potential. In H. Forgasz et al. (Eds.), *Research in mathematics education in Australasia 2004–2007* (pp. 165–193). Rotterdam Netherlands: Sense.

Towers, J., & Davies, B. (2002). Structuring occasions. *Educational Studies in Mathematics, 49,* 313-340.

Walshaw, M. (2004). A powerful theory of active engagement. *For the Learning of Mathematics, 24*(3), 4-10.

Walshaw, M., & Anthony, G. (2008). The role of pedagogy in classroom discourse: A review of recent research into mathematics. *Review of Educational Research, 78,* 516-551.

Watson, A. (2002). Instances of mathematical thinking among low attaining students in an ordinary secondary classroom. *Journal of Mathematical Behavior,* 20, 461–475.

Watson, A., & De Geest, E. (2005). Principled teaching for deep progress: Improving mathematical learning beyond methods and material. *Educational Studies in Mathematics, 58,* 209–234.

Watson, A., & Mason, J. (2006). Seeing an exercise as a single mathematical object: Using variation to structure sense-making. *Mathematical Thinking and Learning, 8,* 91–111.

Wiliam, D. (2007). Keeping learning on track. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1053-1098). Charlotte, NC: Information Age.

Zevenbergen, R., & Lerman, S. (2008). Learning environments using interactive whiteboards: New learning, spaces or reproduction of old technologies. *Mathematics Education Research Journal*, *20*(1), 107–125.

.

Authors:

Glenda Anthony
Massey University, New Zealand
Email: g.j.anthony@massey.ac.nz

Margaret Walshaw
Massey University, New Zealand
Email: m.a.walshaw@massey.ac.nz