

# European and Chinese Cognitive Styles and Their Impact on Teaching/Learning Mathematics

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*This paper tries to provide evidence that research on the cognitive studies should take into consideration the socio-cultural context playing an important role in student learning Mathematics. Through a linguistic-cultural approach the paper wants to be a further close examination on the Didactic thematic related to a possible comparison between East and West and, particularly, between Chinese and Italian mathematical thoughts in some particular aspects related to the phase of argumentation and proving. A discussion of the two countries cultural coordinates and the reference to the Experimental Epistemology permit an overview of some experimental works conducted by the G.R.I.M. of Palermo.*

**Key words:** teaching/learning mathematics, cognitive styles, experimental epistemology.

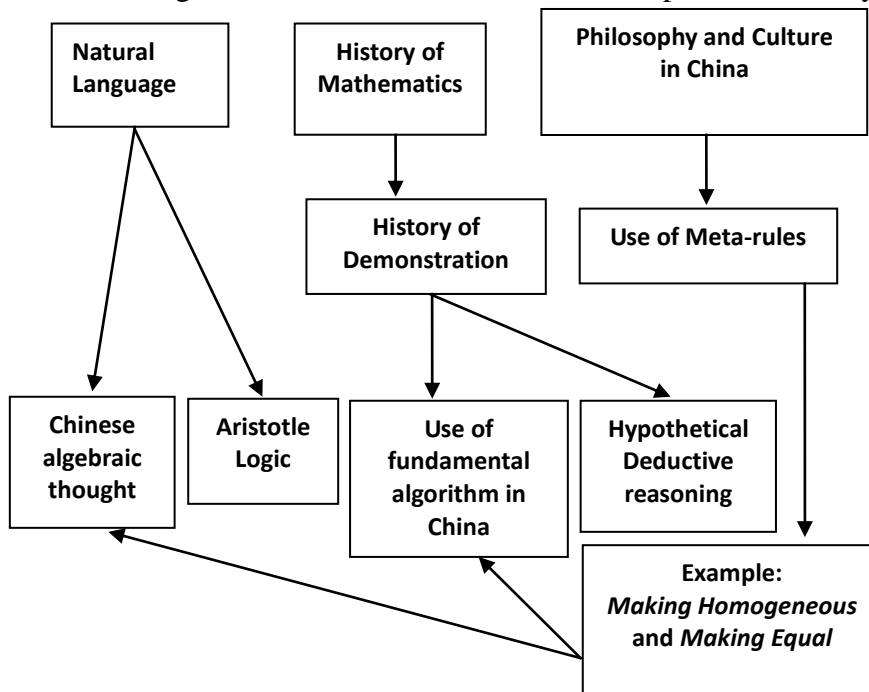
## **Introduction**

To study the phenomena of Teaching/Learning of Mathematics in a globalized world requires a very detailed analysis. We have to consider many *cultural elements* that are present in classes in order to better observe similarities and differences between the processes involved by the students of different cultures. This could help us to understand, describe and possibly anticipate their behaviour and their difficulties in our class. To do this we need to develop survey theoretical-experimental tools, where the relationship between theory and the experimental practice is dialectical, according to a Western view, while with different setting in a relationship with a Ying/Yang theory, according to the East culture. The G.R.I.M. (Group of Research on Teaching/Learning of Mathematics) research group of the University of

Palermo, worked for many years in this direction and tries to keep the following principles: *to try to see/observe and to study the Didactic phenomena from the perspective of other cultures, it is necessary to underline almost all the cultural tools as natural language, History of Mathematics, Epistemology of Mathematics, Neuroscience, etc.* According to this point of view, by studying and analysing the Chinese written language, the History of Chinese Mathematics, the behaviours of Chinese students in Italy, etc. we were able to better understand our system of cultural reference; we experimentally realized that we are Aristotelian and this might affect our way of argue and conjecture in Mathematics but not only. (Paola, 2010; Spagnolo et al., 2009). Other consideration should be underlined for different cultures.

### **Cultural Coordinate to Define a Possible Theoretical Framework: the Experimental Epistemology**

The following schema puts in evidence some interesting cultural coordinates taking into consideration our theoretical/experimental analysis:



*Schema 1. East-West cultural coordinates*

The *Experimental Epistemology* as theoretical framework, implied under the schema, is be considered, according to us, as a principal reference for this

kind of difficult research work and as an important tool to understand the multicultural phenomena of Teaching/Learning of Mathematics.

In the perspective of research of the trials of the Teaching/Learning, the expression *Experimental Epistemology* explains in a good way our point of view putting in evidence the "theoretical-experimental" modelling of the nowadays complex phenomena of Teaching/Learning. In this sense, the used expression must be seen as a dialectical relationship between the two components (*theory* and *Didactical practice*) and it represents, therefore, the overcoming of this opposition in the sense of the dialectical Hegelian. The research in Didactics of Mathematics should have to keep in mind the references of Epistemology and History of the discipline and the references to Cognitive aspect, as well as the references to today's Neurosciences; trying to interpret the complex *dynamics* of the classes. (Brousseau, 1997; Spagnolo et al., 2009; Spagnolo & Di Paola, 2010). The expression *Epistemology* takes back what had been elaborate in the Teaching and in the Learning in the past and, as we said before, it could be useful to interpret the phenomenon of nowadays Teaching/Learning in intercultural class. This will be broadly augmented in the text in a specific manner.

### **The Differences and the Analogies of the East-West Cultural Coordinates**

An analysis of the *cultural coordinates* retained today foundations of Western thought and Chinese Oriental thought, allows us to be able to define a first classification of the fundamental disciplines (*Arts*) in the two cultures. Classification that is documented above in text, even if in a first step, in a detailed relationship with *Writing* and *Wei-ch'ì, Rhythm and Music* that are strongly culturally connected to the philosophical circle, the Science and Mathematics of Chinese and Western thought. Above we report a summarizing table of the classification of the *Arts* in the two cultures:

*Table 1*  
**East and West Art Classifications**

<b>Chinese Classification</b>	<b>Western classification</b>
<b>Music</b> , Discipline related to hearing and also to sophisticated processes that regulates vibrations. It is the practice of the <i>inner qi</i> .	Trivio ( <i>artes sermocinales</i> ):

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It is referred also with embody musical rhythms. Today we know that it is related to natural language and numbers Learning.

**Wei-ch'I,** Relationship between partial modelling and holistic view with a strategic intention.

**Writing,** Holistic representation, algebraic structure, concept of variable and parameter (In a Mathematical sense).

**Painting,** It is considered to be a quiet and direct discipline, accessible to all.

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**Grammar, Rhetoric  
and Dialectics**

Quadrivio (*artes reales*):

**Arithmetic, Geometry,  
Astronomy and Music**

### An Example of Experimental Works

In our schools, defined nowadays in a multicultural setting, to analyse the cognitive styles of students through a socio-cultural approach, highlighting potential similarities and differences in *patterns* of thinking, attitudes, beliefs and conceptions of various ethnic student' groups with respect to the acquisition of specific mathematical concepts, is certainly a complex didactical and research activity but is now a "need" of teachers, researchers and students themselves for a possible integration and mediation of knowledge.

The national and international research in Mathematics Education in recent years has shown sensitivity to the treated problems. Possible "comparisons" between strategies of Teaching/Learning used in different countries (mainly for Primary Schools) were defined according to some key contents in Mathematics such as Arithmetic and Algebra (Cai & Nie, 2007; Leung, 2008). The literature has also highlighted proposals in education capable of fostering the integration of pupils of different ethnic groups in the classes of the host through the establishment of intermediary cultural environments (Favilli et al., 2003).

Over the past ten years we have also witnessed an increase of more and more research on the judgment between the mathematical performance of students attended different school grades; particularly significant were those aimed at students of U.S. and Asian countries such as China, Japan and Korea (An, 2008; Cai & Nie, 2007; Cao et al., 2002) while few are the research work in Italy and, specifically, works that take into account the nowadays inner setting of the Italian classroom.

Our recent research, referred to this specific subject, aims to analyse

some of the possible similarities and differences in the forms of logical argue reasoning highlight by Chinese and Italian students attending Italian schools (but not only) at all school levels. We try to do this kind of investigation trough an attentive observation of European and Chinese cognitive styles and their impact on Teaching/Learning Mathematics. The generalization of the results is ensured by the method of *Experimental Epistemology* and through a careful a-priori analysis of the behaviour of the students (Brousseau, 1997) that takes into account the cultural processes and an analysis of the phenomenon of Teaching/Learning both quantitatively and qualitatively. This analysis is conducted in a continuous balancing from micro-Didactic/macro-Didactic. Essentially, a detailed micro analysis may allow us to hypothesize inferences on the macro.

The conducted research work is referred primarily to four issues related to each other and developed through experimental investigations related to theory/practice. The first refers to an experimental work (Ajello, Spagnolo, & Xiaogui, 2005; Spagnolo et al., 2009) conducted in parallel with Palermo and Nanjing (China); the second refers to a PhD thesis conducted in a multicultural classroom in Palermo (Di Paola 2009). The third part of the research project conducted on this subject, specifically refers to the view of Education in the European and Chinese culture (Spagnolo & Di Paola, 2010). It is discussed on this paper on the sixth paragraph. Finally, the last table of this paper presents, even if very briefly, the last part of our research work aimed to study the mathematical aspects of Rhythm, Scale and Metric in the two considerate cultures from a scientific point of view. We try to find possible links with the Teaching/Learning phase of Mathematics in Chinese and Italian class (Spagnolo & Di Paola, 2010).

The research topic refers specifically to logical argumentation and conjecture, trough different experimental works conducted in classes in Arithmetic, Algebra and Geometry; we aim to present situations/problems typical of mathematical thought with particular attention to logical and linguistic problems and argumentation of the students. The most significant reference of the research is the survey on patterns of reasoning in relation to Eastern and European historical epistemology. Equally important for the study of contingencies is the experimental analysis on the use of epistemic logic with respect to the underlying Natural Languages of the students involved.

The main hypothesis of the research was so defined as follows:

*H1: The differences and similarities in the History of Eastern culture and Western heritage also have an equivalent in the differences and*

*similarities between the patterns of reasoning found today in the Teaching/Learning of Mathematics.*

More specific hypotheses were fixed after this one, taking into account the specificity of the arithmetic/algebraic/geometric thoughts in the two cultures. In order to falsify the hypothesis of research, the following paradigmatic references are used:

1) Historical analysis and historical-epistemological mathematical thinking, as regards the study of differences in the forms of reasoning (to argue, to conjecture and to demonstrate) and the algebraic tradition in the History. The latter is read through a "comparison" between the *Elements* of Euclid, as a canon reference to the Western tradition Mathematics and *Jiuzhang Suanshu* for the Chinese. The references are mainly theoretical in this sense of the work of Chemla (2003, 2004), Needham (1962), and Granet (1988);

2) Natural Language analysis and Neuroscience in alphabetic and ideographic languages as a comparison of the cognitive aspects related to the Natural Language of origin of students and thus may have impact on the Learning of Mathematics. We study the possible meta-rules as a settlement of the Chinese written language in relation to some cultural aspects of Chinese culture as the Taoist yin-yang and some aspects that could be found as scheme patterns in algebraic pre-algebra reasoning;

- Experimental analysis of situations/problems through the approach of the Theory of Didactic Situations (Brousseau, 1997; Spagnolo & Malisani, 2008);

3) Analysis of cases through the individual interview method.

For a detailed analysis of the comparative study between Chinese thought and Italian, in situations of Teaching/Learning in multicultural perspective, we refer, in a first approximation, to the studies of D'Ambrosio (1992), Bishop (1998) and Nisbett (2007).

### **An Overview of Some Experimental Results**

Regarding the first issue, the data collected (Ajello, Spagnolo & Xiaogui, 2005; Spagnolo et al., 2009) seem to confirm that Chinese students conduct a highly pragmatic, concrete, process linked to a fundamental algorithmic procedures closely related to a holistic thinking of encoding and decoding information in different mathematical situations/problems presented. These, in the process of mathematical reasoning and conjecture, generally show, according also to the results of the research conducted by Di Paola (2009), a heuristic trial and error procedure to find a *basic algorithm* as a tool of

demonstration. For Chinese students, the typical way to argue and to organize the different steps of solution of Arithmetic and Algebraic problem solving, is for a hierarchy of models of reasoning (and sub-models as sets and subsets) that seems to refer to implications Fuzzy (Gras et al., 2008). The typical reasoning shown by Italian students is rather a hypothetical-deductive reasoning through a chain of finite conjunctions contextualised to a bivalent logic. Using written Chinese language from a theoretical prospective, in Di Paola (2009) we tries to provide a particular approaches for understanding student Algebra cognitive development, paying special attention to an epistemological and linguistic view of the passage between Arithmetic to Algebraic thought in Chinese and Italian students.

The two different views have been aligned with the appropriate historical epistemological analysis of Mathematics in the two cultures with particular reference to this subject and “demonstration” (Needham, 1962). The most significant obtained results can be briefly summarized in the following table:

*Table 2*

**Prevalent Behaviours in the Experimental Results**

<b>Prevalent behaviours in the experimental results in Italy</b>	<b>Prevalent behaviours in the experimental results in China</b>
<i>Inductive reasoning: finite chain of conjunctions.</i>	<i>Heuristic approach for attempts and errors. Research of an algorithm as a tool of formalised demonstration. Experiment and induction. Proof and intuition.</i>
<i>Use of Venn’s diagrams for deduction</i>	<i>Request of a concrete context to analyse the “adequacy” of the proposition. Initial proof of truth and falsity of a proposition by trial and error.</i>
<i>Correct interpretation of syllogisms. Deductive processes in N.L.</i>	<i>Attention to the analysis of the written text. Use of tables or matrices for the data. Correct us of different reasoning patterns found on the definition of the problem.</i>

**Strategic Thinking with Chess and Weich’i: Possible Links with Education in the Two Cultures**

According to the literature discussed on the construct vision of

*Education* in China and European countries (according to the Ministries of Education), we realized that in Western cultures the meaning of *strategy* and *tactics* are almost retrieved from the hypothetical-deductive reasoning.

As in Mathematics and Teaching Mathematics, the *Euclidean Geometry* and the relevant thought implied under it, is a model of Aristotelian logic and it has been the model of reference for about 2000 years; not only in Mathematics. For example, in Western culture, we have as interesting reference for strategic thinking the game of Chess, totally different from the Chinese chess (*Weich'i*) on which it is possible to observe the different Eastern reference for strategic thinking. With a sort of comparison between Western and Chinese chess we realized that the logical schemes developed on Chinese chess may represent strategic thinking in the Chinese cultural tradition. Strategic thinking that is interesting also for the Education point of view. The table that follows, with the limit of any diagram, may be helpful to focus those elements that can then be used for any educational issues:

*Table 3*  
**Chess and Wei-ch'i.**

<b>Discipline</b>	<b>Purpose of the game</b>	<b>Social function</b>	<b>Strategic elements</b>	<b>Function of tactics</b>
<b>Chess</b>	Checkmate (Capture of the Adversary King).	None in particular – an exception is the Soviet Union from 1925 to about the 80s	Identification of partial objectives and evaluation of the position. Optimisation of the action of one's own pieces and limitation of the adversaries. Recognition of visual and abstract patterns. <i>Concept of</i>	Determining. A single tactical action, well carried out, becomes the main and often conclusive one of the match. The



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<b>Wei-ch'i or Go</b>	Control of the largest area possible of the territory on the goban.	Very important – second place amongst the traditional arts and held to be necessary for the education and the instruction of functionaries and dignitaries in imperial China and imperial Japan	<i>control</i> , in the sense of a continuous evaluation of the total situation. Choice of that which can be held to be satisfactory. Strategic items: defence of own groups, attack to opponent's groups, territorial gain.	knowledge of the tactical themes is very important, but a local victory risks being lost. Tactics are not tied to strategy. The concept that a tactical success leads to a strategic success is alien to the spirit of Wei ch'i.
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We maintain that several themes explained above can have connections to Didactics subject. The different approaches to *strategy* and *tactics* in different cultures, supply, in their entirety, some formidable tools for confronting the most disparate themes of life and are very interesting in a didactic context analysed by quantitative and qualitative methods.

Some schematic notes on these possible connections are reported below:

1) First point, the full awareness that without *dedication*, unity of intent, clear definition of the goal, attention and motivation it is rare that one reaches an objective whatever it is. This point is of a general character, but precisely for this it is often neglected in Didactics. Demotivation and too bureaucracy is the probable reasons of it.

2) The aptitude of *Evaluation*: the considering of the elements with an elevated degree of objectivity and selectivity of one's attention allows the formulating of strategies, and sometimes also methods and tools capable of the solution of problems and of the definition of a situation. That in line with a modern vision of Didactics which aims at an increase of competences, i.e., of the use of one's knowledge and abilities in different contexts and situations;

this particularly in Italy, where there is a proven lack in this direction.

3) *Hypothetic-deductive thinking*, also at a high deepening level, it is an important item in strategy games and useful in all fields. Hypothetic-deductive thinking is found in the western Culture since from Euclide's Elements (Model of Aristotle Logic). Hence, in the Western culture, strategic thinking may be addressed by Aristotle Logic (inherent to Natural Language), while in the Eastern Culture, as we mentioned earlier, by wei-ch'i "meta-rules".

4) *Visual or abstract patterns recognition*, and continuous adaptation to the reality is the normal practice in chess and in wei-ch'i, typical of high order competences. A strategy adopted in a repetitive way, without adaptation to reality, can be proven wrong by practice. This is related to Epistemological Obstacles Theory in Education. This feature is found in both cultures; chess and wei-ch'i support it.

5) *Education of strategic and tactical features recognition could be important in social fields*. It could become, as mentioned earlier, a useful tool for understanding different cultural approaches. We can imagine a more Hypothetic- deductive approach by Western students, and a sharp separation between strategy and tactics. In the Eastern culture, this separation is not delimited. Probably this approach would depends also on Weich'i meta-rules. These meta-rules may clarify the differences between strategy and tactics in a more subtle way with respect to the Western approach (Spagnolo et al., 2005)

6) The separation of self-concept being separated from others as a reference for decisions, and as a *constant search for best solutions* is useful topics in Education.

### **Conclusion and Further Studies**

According to the discussion presented on the paper, the significant results that we believe may be instruments of observations in multiculturalism Chinese classes in Mathematics (but not only) are diverse a with a large spectrum. We try to summarize these in the following table. The last part of the table discuss also some consideration on the first Chinese cultural Art on which we are working on, studying it form a scientific point of view in relation on the mathematical aspect of Rhythm, Scale and metric in the two considerate cultures.

*Table 4*

#### **Possible Cultural Instruments of Observations in Multiculturalism**

### Chinese Classes

	European	Chinese
<b>Natural Language</b>	Alphabetic writing.	Ideographic writing categorized with meta-rules.
<b>Used logical schema</b>	Lateral thinking, influence of different logic in the Mathematics argumentation phases: 1. Bivalent logic (about 80%) 2. Fuzzy Logic (about 20%).	Relational associative thinking, influence of different logic in the Mathematics argumentation phases: 1. Bivalent Logic (about 20%) 2. Fuzzy Logic (about 80%).
<b>History of argumentation and demonstration</b>	Aristotle, Plato, Hegel and the hypothetical-deductive reasoning. Euclid as a main paradigm of Mathematics and as a Logical-argumentative schema's system.	Tao, Confucius, Buddha, fundamental algorithm for classes of problems (according to the Jiuzhang Suanshu) defined on meta-rules.
<b>Strategy</b>	The hypothetical-deductive system as a strategic winning system with the use of patterns of recognition.	Concept of control, in the sense of a continuous evaluation according to fuzzy logic.
<b>Music</b>	Rhythm and syllables, Rhythm and Metrics, Metrics such as succession of syllables Scale: modal, tonal, dodecaphonic  Rhythm as an order of movement (Plato)	<i>čie</i> (rhythm), <i>pan-yen</i> (measure) the rhythm track (measures 2 and 4 times) is frequent; the ternary is rare in traditional classical music. Scale: pentatonic The music is nature which is manifested

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	through the sound and the rhythm a "look" of the sound
Middle Age: Gregorian with free rhythm and meter without quarrels pre-stability	Language monosyllables Singing syllabic
Music and text: Italian Melodrama	Music lyrics: Italian Opera Music lyrics: melodic compositions and polyphonic harmony without the Western
Measured rhythm: is based on the provision of accents. Free rate (no provisions in the accents) and measured	Free rhythm (no provisions in the accents) and measured

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We think that these results can be generalized to different cultural backgrounds and we believe, in this sense that experimental investigations carried out directly in China and Italy would allow further research on Natural Language and mathematics argumentation that may result from it.

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