Teachers Attending to Students' Reasoning: Using Videos as Tools

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This paper describes an intervention model for the professional development of middle-school classroom and special education teachers that makes use of videos of children's mathematical reasoning drawn from longitudinal and cross sectional research. The video and related data are made available through the Video Mosaic Repository of Rutgers University. A goal of the intervention was to stimulate a change in teacher beliefs about children's learning by providing video examples of children's reasoning under conditions that support learning. During the yearlong intervention, teachers replicated instances of the video learning environments in their classrooms and analyzed the reasoning of their own students. To assess teacher changes in beliefs about learning and teaching during the intervention, pre- and post-test beliefs were administered. Analysis of data shows significant changes in teacher beliefs about learning and teaching.

Key words: teacher development, middle school, video repository, counting and combinatorics.

Introduction

Based on an extensive program of longitudinal and cross-sectional research, now in its 24th year, we have been following the collective building of mathematical ideas and ways of reasoning by learners. The multiple studies produced a large collection of video and related metadata, demonstrating students' reasoning and enabling one to trace the development of reasoning of individual and groups of students over many years, elementary through high school, and in some cases beyond. The studies have produced a unique collection of video and related data that are available on the *Video Mosaic*

Repository [1] at Rutgers University (Agnew, Mills, & Maher, 2010). The Repository is an outgrowth of our NSF funded research and development project: Cyber-Enabled Design Research to Enhance Teachers' Critical Thinking Using a Major Video Collection on Children's Mathematical Reasoning.[2]

The Video Mosaic Repository

The Video Mosaic Repository was designed to preserve the unique video collection amassed by The Robert B. Davis Institute for Learning at Rutgers University through more than two decades of research with over four millions dollars of grant funding from the National Science Foundation.[3] In addition to preserving the video collection, new tools were developed for conducting design research with empirical studies that use the videos in the context of teacher education. The collection of videos stored in the Rutgers Repository provides an important resource for pre-service teacher preparation and for professional development with practicing teachers (See, for example, Maher, Palius, & Mueller, in press.). The video and related data in the Repository show the reasoning of students from elementary through high-school years, and in several content strands, where it is possible to search the collection and follow particular students investigating mathematics within and across strands. For this paper, we illustrate how we integrate the video collection with our professional development work with middle-school grade teachers. Our examples come from the videos of the counting/probability strand and extend through the middle-grade combinatorics strand. We focus on the forms of reasoning exhibited by the children in their justifications of solutions to

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[1] Note that the video clips that we refer to in this paper, as well as others from the collection, are available on the Video Mosaic Repository, which is located at: http://www.video-mosaic.org/

problems that are later described.

Theoretical Perspective

Our approach to professional development is based on the view that there are necessary prerequisites in teacher knowledge that need to be in place in order for teachers to learn how to attend to the developing ideas and evolving growth in mathematical reasoning of their students. These prerequisites include a deep knowledge of the underlying mathematics that is taught, how students learn the mathematics, and how classroom environments can be designed to motivate and support children's learning. Our research has shown that individual learning manifests itself through the social interactions of others (Weber, Maher, Powell, & Lee, 2008). In the activity of problem solving, learners build and share ideas, and in so doing, deepen and extend their knowledge (Davis & Maher, 1997; Maher, Martino, & Alston, 1993). Within a learning community, individuals have access to the ideas of others. Ideas are interconnected and extended as learners work together to make sense of each other's ideas and build convincing arguments for their solutions to problems (Maher, Powell, & Uptegrove, 2010). To create a classroom environment that reflects this perspective, a new view of learning is required. It is important that teachers believe that students are capable of thoughtful mathematical reasoning. Also, they need to recognize the key role they play in designing classroom conditions for children's learning.

Objective

The professional development intervention is part of a multifaceted research and development project that seeks to preserve and make accessible videos from the unique collection made possible by research from The Robert B. Davis Institute for Learning at Rutgers University. The collection includes video data that span more than two decades of research across grade levels, schools, and content domains. This paper, based on an earlier version presented at Hangzhou Normal University[4], describes how the videos from the Rutgers Repository are integrated into a professional development program for

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[4] An earlier version of this paper was presented at The 13th International Conference on Mathematics Education in China (ICMEC-2010), June 25-28, 2010, Hangzhou Normal University, Hangzhou, P.R. China.

teachers in order to provide some insight into how teachers' beliefs about learning and teaching are influenced by studying videos of children's learning. The videos illustrate children engaged in mathematical problem solving and offering justifications for their solutions.   Hence, our research explores the impact of studying videos of children's reasoning on teacher beliefs.

The Professional Development Program: A Model

In order for teachers to follow the mathematical reasoning of their students, they too must be capable of providing convincing arguments for the solutions to problems. An important prerequisite of our professional development work with teachers is to improve their mathematical reasoning skills so that they are better prepared not only to study the videos of children's reasoning, but also to promote and evaluate the mathematical reasoning of their own students.   Our approach is rooted in prior work that yielded the Private Universe Project in Mathematics, a series of six video workshops with an accompanying guide (Maher et al., 2000).

Our Approach

There are three intervention cycles that occur over several months, and each cycle has four components: (1) *teachers doing mathematics*, (2) *teachers studying videos of children doing mathematics*, (3) *teachers implementing in their classrooms*, and (4) *teachers analyzing their students' work*.   First, we describe the components and then we provide examples through the cycles of implementation.

Teachers doing mathematics. Within each cycle, new tasks are introduced. Across the cycles, tasks become increasingly more complex. The problems come from a strand of tasks that were used in research on children's reasoning (Maher, Powell, & Uptegrove, 2010). They are designed to offer opportunities for teachers and their students to make connections between and among problems of similar structure and, when appropriate, to pose generalizations for the solutions. For each task, teachers work in small groups to develop solutions and convincing justifications. After working on the tasks, the variety of solutions are shared and discussed in workshop format. Attention is given to multiple approaches and varieties of forms of reasoning.

Teachers studying videos. Also, within a cycle, teachers study videos of students working on the same tasks and under similar conditions. Emphasis is placed on analyzing the forms of reasoning exhibited by the students in the videos. The videos that are studied show impressive work of students and give striking attention to the powerful reasoning of children across ages, elementary through high school, in multiple settings. We view them as an important tool in helping teachers become more aware of the potential for mathematical reasoning of students.

Teachers implementing in classrooms. Continuing with the first cycle of doing mathematics and studying videos, teachers implement the same task in their own classrooms with the students they teach. When possible, teachers arrange to observe the implementation of their colleagues. This component takes place over a few weeks.

Teachers analyzing student work. The cycle is completed with an analysis of the forms of reasoning exhibited in the written work of their students. By carefully analyzing and discussing the work of their students, teachers express enthusiasm that their own students are capable of producing reasoning similar to theirs and similar to the children in the videos.

The Intervention – Some Examples

The first cycle begins with a daylong workshop, whenever possible, to immerse teachers in the process of becoming active participants in a learning community. The facilitator engages teachers by inviting them to work in small groups to build a solution to the following task:

Cycle 1, task 1. *You have two colors of unifix cubes available to build towers. Your task is to make as many different looking towers as possible, each exactly four cubes high. Find a way to convince yourself and others that you have found all possible towers four cubes high, and that you have no duplicates.*

Teachers are provided with two colors of unifix™ cubes and papers and colored markers to record their solutions. Figure 1 shows teachers building and discussing their models of the 4-tall tower problem, selecting from two colors during the problem-solving workshop.

Figure 1. **Teachers working on tower problems.**

After working on the task for approximately one-half hour, small group solutions and arguments are shared. Figure 2 shows teachers sharing their solutions to the 4-tall tower problem using an argument by cases.

Figure 2. **Teachers sharing tower groupings.**

When the sharing of solutions and convincing arguments have been completed, teachers are then introduced to the first video.

Cycle I, video 1. This video, *Stephanie and Dana, Grade 3,* shows Stephanie and Dana working together building 4-tall towers, selecting from two colors, and finding a total of sixteen.

During the workshop, the facilitator initiates a discussion about the third graders' problem solving and that of the teachers. The teachers are thus enabled to point to comparisons between heuristics and strategies that have been used by both the teachers and the children; similarities that may be

surprising to teachers are noted. Teachers are then presented with the following extension problem to building towers, selecting from two colors:

Cycle 1, task 2. *Make a prediction about a solution for finding all possible towers 3 cubes high (without building them). Do you think there will be more, fewer, or the same number of possible towers as you found for towers that were 4 cubes high?*

After making their predictions, teachers study two videos.

Cycle I, video 2. This video, *Meredith Removes the Top Cube*, shows an interview of third grader, Meredith, about her problem solving with her partner, Jackie. In an earlier classroom session on the 4-tall tower problem, selecting from red and yellow cubes, Meredith and Jackie produce an argument by cases to justify the sixteen towers (Maher, 2009)**.** The video shows the interview that followed with Meredith, who was asked to *predict* the number of towers there would be for 3-tall towers. She predicts that number would be the same number, arguing that if one removed a cube from the top of all of the sixteen, 4-tall towers, there would remain sixteen 3-tall towers. The interviewer asks Meredith to investigate her claim. As she does so, Meredith recognizes pairs of duplicate towers and explains why duplicates should be eliminated, then changes her answer to eight towers.

Cycle I, video 3. This video, *Stephanie Elaborates on Her Prediction,* follows the 4-tall tower building work of Stephanie and Dana as third graders. They have been asked to conjecture how many 3-tall towers could be built, selecting from two colors. They conjecture that there would be more because removing a cube from the top of all of the 4-tall towers would provide extra cubes for building more towers, and are then encouraged by the researcher to investigate. In the video, Stephanie discusses her ideas with the researcher and modifies her prediction with an explanation.

Teachers are then presented with the following extension problem to building towers, selecting from two colors:

Cycle 1, task 3. *Make a prediction for finding all possible towers that are 5 cubes high (without building them). Do you think there will be more, fewer, or the same number of possible towers as you found for towers that were 4 cubes high?*

After making their predictions, teachers are provided with unifix™ cubes and asked to build 5-tall towers, selecting from two colors.

Cycle I, task 4. *You have two colors of unifix cubes to build towers. Your task is to make as many different looking towers as possible, each exactly five cubes high. Find a way to convince yourself and others that you have found all possible towers five cubes high.*

After working on the task for approximately 45 minutes, small group solutions and arguments are shared. This task tends to elicit reasoning by cases, although sometimes an inductive argument is provided. Teachers then study another video.

Cycle 1, video 4. This video, *Stephanie and Dana, grade 4*, features the girls as seen before, but now as fourth graders, engaged in building 5-tall towers when selecting from two colors. Working together on the task, they reach a solution with 32 towers.

The teachers then discuss the girls' problem solving shown on the video, making comparisons and contrasts with what the girls do. Teachers discover that they identify the same kinds of patterns and groupings that children working on this task also identify and name, such as *elevator*, *staircase*, *opposites* and *cousins* (Maher, Sran, & Yankelewitz, 2010a). Teachers are then shown another video, featuring Milin sharing with some of his fifth-grade classmates an inductive argument he initially had built as a fourth grader (Maher, Sran, & Yankelewitz, 2010b)

Cycle 1, video 5. This video, *Milin Shares His Inductive Argument*, shows several children, now fifth graders, working on a new task that gives opportunity to extend their thinking about the towers. The video illustrates how ideas can travel in a classroom through the sharing of an inductive argument for building towers of any height, *n*-tall, selecting from two colors.

Cycle 1, classroom implementation. After doing mathematics and studying videos, the teachers are well prepared to engage their own students on the 4-tall towers task, selecting from two colors. As the children work in small groups on the problem in their classroom, teachers listen to children's explanations in order to follow their reasoning. Figure 3 shows a sixth-grade teacher examining the tower models built by two of her students.

Figure 3. **Teacher studying students' models.**

Cycle 1, teachers analyze students' work. After teachers implement the task in their classrooms, they bring samples of their students' work to share and analyze with their colleagues at the next workshop. Students' work tends to display a variety of representations and notations, as well as multiple forms of reasoning. Figure 4 shows the sixteen-tower solution of a student who represented the towers with B (blue) and Y (yellow) symbols. Three groups are illustrated: all one color, two of each color, and exactly one (and 3) of a color.
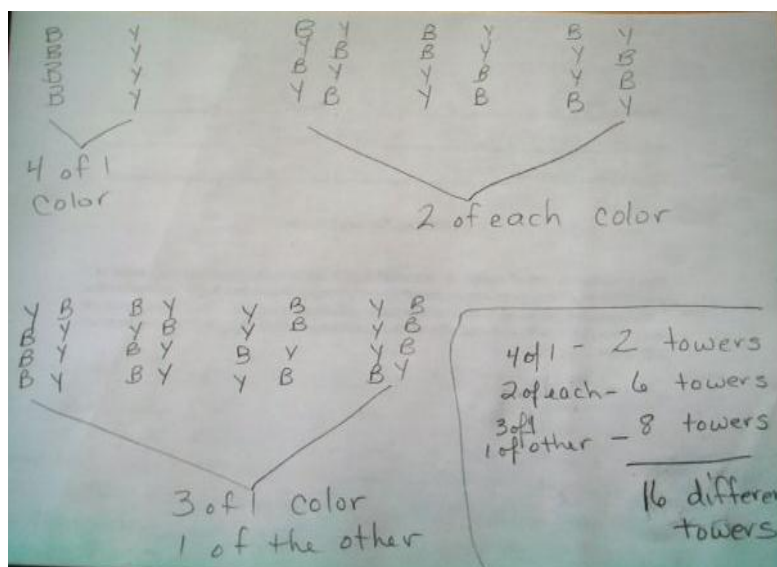

Figure 4. **Teacher sharing student notation.**

Figure 5. Shows a student's use of tower drawings and symbols to indicate groupings similar to those illustrated in Figure 4.
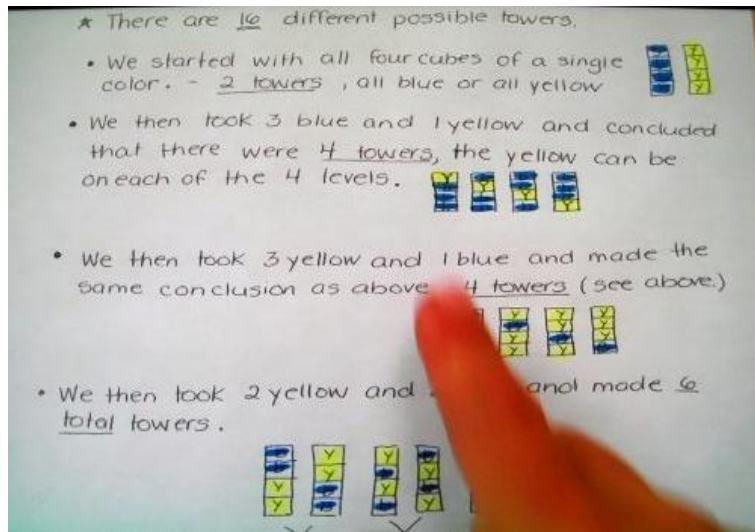
Figure 5. **Teacher sharing student groupings.**

Cycles II and III follow a similar structure to Cycle I, with different tasks and videos. For the second cycle, the tasks are a class of pizza problems; that is, determining how many pizzas it is possible to make when selecting from various number of toppings and under a variety of constraints. Based on findings from the longitudinal study about the representations that students created and the forms of reasoning that emerged when students had to grapple with the complexity of pizza with halves (Maher, Sran, & Yankelewitz, 2010c), we suggest introducing the pizza problems in the following sequence:

Cycle II, task 5. *A local pizza shop has asked us to help them design a form to keep track of certain pizza sales. Their standard "plain" pizza contains cheese. On this cheese pizza, one or two toppings could be added to either half of the plain pizza or the whole pie. How many choices do customers have if they could choose from two different toppings (sausage and pepperoni) that could be placed on either the whole pizza or half of a cheese pizza? List all possibilities. Show your plan for determining these choices. Convince us that you have accounted for all possibilities and there could be no more.*

Cycle II, task 6. *The local pizza shop was so pleased with your help on the first problem that they have asked us to continue our work. Remember that they offer a cheese pizza with tomato sauce. On this cheese pizza, one or more of the following toppings could be added to either half of the plain pizza or the whole pie: peppers, sausage, mushrooms, and pepperoni. How many choices does a customer have? List all the possible choices. Find a way to convince each other that you have accounted for all possible choices.*

Cycle II, task 7. *Capri Pizza has asked you to help design a form to keep track of certain pizza choices. They offer a standard "plain" pizza with cheese and tomato sauce. A customer can then select from the following toppings: peppers, sausage, mushrooms, and pepperoni. How many choices for pizza does a customer have? List all possible choices. Find a way to convince each other than you have accounted for all possibilities.*

After the teachers engage in doing mathematics through these tasks, there is video to be studied of children working on the pizza problems and talking about how to solve them.

Cycle II, video 7. This video, *Exploring Pizza Problems in Grade 5*, show fifth grade students working on the pizza problems over several classroom sessions. There is much debate as they search for good ways to represent the various options for pizzas with halves when selecting from two toppings. They struggle with this complicated task, yet their work on it gives them insights for approaching the other pizza problems that they do next.
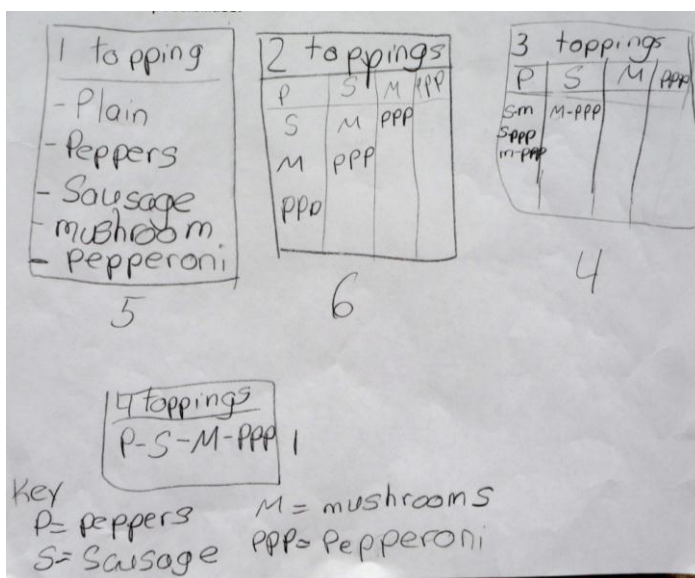
Figure 6. **Teacher sharing student solution to pizza problem.**

Similar to the first cycle, after doing and discussing their solutions to the problem with their colleagues and watching videos of children doing the same problem, teachers return to their classrooms and engage their own students in the same task. Later, they return to a follow-up workshop with samples of their students' work. Figure 6 shows a teacher sharing the argument of Maggie

and Sam, two middle school students. They organize their pizza choices using a case argument for 1, 2, 3 and 4 toppings.

Another example of student work shared by a teacher is given in Figure 7. To account for all 16 choices of pizza, a 0-1 notation was used in table format to record the absence or presence of a topping. Following discussion of students' work, teachers study another video.

choices. Find a way to convince each other that you have accounted for all possibilities.

	Peppers	Sausage	Mushrooms	Pepperoni
✓1.	1	1	1	1
✓2.	1	0	1	1
✓3.	1	0	0	1
✓4.	1	0	0	0
✓5.	0	1	1	1
✓6.	0	1	1	0
✓7.	0	1	0	0
✓8.	0	0	1	1
✓9.	0	0	1	0
✓10.	0	0	1	0
✓11.	1	0	1	0
✓12.	0	1	0	1
✓13.	1	1	0	1
✓14.	0	0	0	1
✓15.	1	1	0	0
✓16.	1	1	1	0

Figure 7. **Teacher shares students' zero one notation for pizza problem.**

Cycle II, video 8. This video, *Brandon Invents Isomorphism*, is an interview with 10-year old Brandon that is conducted after he has worked with a partner in the classroom on some problem-solving tasks, most recently the 4-topping pizza problem. Brandon uses a 0-1 notation to account for all pizza choice. When the researcher asks Brandon if this problem reminded him of any others, he refers to the towers problem, saying it is "the same". He then rebuilds his solution set of towers to demonstrate their equivalence.

The teachers then begin the third cycle and are given Tasks 8 and 9 to work on with a partner or small group. After discussion of their problem-solving strategies and solutions, they watch Cycle III, Video 9. The tasks and video are described below.

Cycle III, task 8. *Find all possible towers that are three cubes tall, selecting from cubes available in three different colors. Show your solution and provide a convincing argument that you have found them all.*

Cycle III, task 9. *Ankur's Challenge: Find all possible towers that are four cubes tall, selecting from cubes available in three different colors so that the resulting towers have at least one of each color. Show your solution and provide a convincing argument that you have found them all.*

Cycle III, video 9. This video, *Romina's Proof*, shows a group of five tenth-grade children working on the Ankur's challlenge task. In the episode, five students begin their work as a group of two and a group of three. The video clip focuses on three students - Romina, Jeff and Brian - who develop a notation to represent the three colors and build an argument that is shared by Romina to the entire group, convincing them of their solution of thirty-six towers.
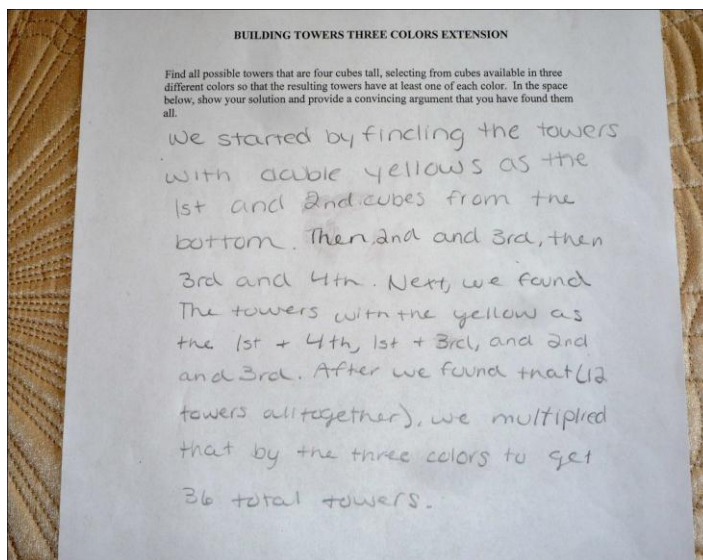

BUILDING TOWERS THREE COLORS EXTENSION

Find all possible towers that are four cubes tall, selecting from cubes available in three different colors so that the resulting towers have at least one of each color. In the space below, show your solution and provide a convincing argument that you have found them all.

We started by finding the towers with double yellows as the 1st and 2nd cubes from the bottom. Then 2nd and 3rd, then 3rd and 4th. Next, we found the towers with the yellow as the 1st + 4th, 1st + 3rd, and 2nd and 3rd. After we found that (12 towers alltogether), we multiplied that by the three colors to get 36 total towers.

Figure 8. **Teacher shares students' solution for Ankur's challenge.**

Again, similar to the first cycle, after doing and discussing their solutions to the problem with their colleagues, and watching videos of children doing this same task, teachers go back to their classrooms and engage their own children in Tasks 8 and 9. They return to a follow-up workshop with samples of student work and share the solutions and arguments used by their children. In our study with middle school teachers, two sixth graders solved the Ankur's challenge problem using a strategy that was similar to tenth-grader Romina, as shown in the *Romina's Proof* video. Their written solution was shared by a teacher as illustrated in Figure 8.

In sum, the yearlong intervention engaged teachers and their students in thoughtful mathematical problem solving and reasoning. A question of interest to us was whether their participation in the intervention had an influence on previously held beliefs about what mathematics children are capable of learning and what role a teacher can have in the process.

Teacher Beliefs Study

It was of interest to investigate whether teacher beliefs about learning and teaching were durable during this intervention. An objective was to track changes, if any, in teacher held beliefs during the course of the intervention. Our expectation was that learning to attend to forms of reasoning they use in problem solving and to be more attentive to children's reasoning by studying videos might affect certain held beliefs. We were particularly interested in whether there would be differences between special education and regular classroom teachers in terms of the expectations about student learning and the conditions that teachers can create to influence children's learning. To this end, a study was designed to investigate whether certain teacher beliefs about children's learning and teaching might be transformed over the yearlong intervention.

Method

Participants

The study consisted of 20 middle school classroom and special education teachers, from two middle schools, in a school district in New Jersey. The regional school district is diverse in the population of students it serves. All twenty teachers participated in a year long, professional-development workshop that was a component of a comprehensive design research study funded by the National Science Foundation[5]. The professional development intervention model is described in detail in this paper.

Instruments

Prior to and after the intervention, the teachers were given a Beliefs Inventory as pre and post-test assessment of their beliefs about children's

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learning and conditions for effective teaching. The Beliefs Inventory included items that assessed beliefs about how mathematics is learned and how teachers influence (or not) children's learning. For this report, we discuss the subset of teacher beliefs that changed during the yearlong intervention (See Table 1). Also, after each session, teachers were asked to evaluate their workshop experience in terms of learning and relevance.

Results

Data from teacher evaluations and Beliefs Inventory give some insight into the value of the intervention. Teacher evaluations after each workshop session were uniformly and consistently positive, suggesting that they found their participation worthwhile and related to their teaching.

Table 1 identifies a subset of 13 of the 34 items in the Beliefs Assessment Inventory that were changed. The table examines post-test belief scores of participating teachers for those items in which the pre-test Belief response was Uncertain, Disagree or Strongly Disagree. It should be noted that, on the average, 64.4% of the participating teacher post-test items indicated growth. In contrast, on the average, only 4.0% of the teacher responses indicated negative growth. For the remaining 32%, on the average, there was no change in beliefs.

Table 1

Post-Test Change in Participating Teacher Beliefs Responses for Beliefs Inventory Items with a Pre- Test Score of Uncertain, Disagree, or Strongly Disagree

Belief Inventory Item	N*	% Post-Test Growth	% Post-Test Decline	% Post-Test No Change
Inverse of: Collaborative learning is effective only for those students who actually talk during group work.	7	100%	0%	0%
Understanding math concepts is more powerful than memorizing procedures.	5	100%	0%	0%
All students are capable of working on complex math tasks.	10	80%	10%	10%
Students are able to tell when their	4	75%	0%	25%

teacher does not like mathematics.				
Learners generally understand more mathematics than their teachers or parents expect	18	66.7%	0%	33.3%
Collaborative groups work best if students are grouped according to like abilities.	18	66.7%	0%	33.3%
Inverse of Young children must master math facts before starting to solve problems.	14	64.3%	0%	35.7%
Math is primarily about communication.	10	60%	10%	30%
Learners generally have more flexible solution strategies than their teachers or parents expect.	12	58.3%	8.3%	33.3%
Teachers should intervene as little as possible when students are working on open-ended mathematics problems.	12	58.3%	16.7%	25%
Inverse of: Teachers should make sure that students know the correct procedure for solving a problem	19	57.9%	0%	42.1%
Inverse of: Math is primarily about learning the procedures.	11	45.5%	0%	54.5%
Inverse of: The idea that students are responsible for their own learning does not work in practice.	9	44.4%	11.1%	44.4%
TOTAL		64.4%	4.0%	31.6%

*N = Number of Participating Teachers who scored Undecided, Disagree, or Strongly Disagree to the Beliefs Inventory Question in the leftmost column

Table 2

Overall Change in Participating Teacher Beliefs for Experimentally Aligned Items on the Beliefs Assessment Inventory

Belief Assessment Inventory Item	Mean Pre-Test Score	Mean Post-Test Score	Mean Growth	Student-t Ratio	Significant Level
Inverse of Collaborative learning is effective only for those students who actually talk during group work.	2.25	1.9	0.35	1.32	0.10
Understanding math concepts is	1.85	1.4	0.45	1.76	0.05

more powerful than memorizing procedures.					
All students are capable of working on complex math tasks.	2.75	2.35	0.40	1.5	0.07
Students are able to tell when their teacher does not like mathematics.	2.0	1.65	0.35	1.5	0.07
Learners generally understand more mathematics than their teachers or parents expect	3.1	2.45	0.65	3.32	<0.01
Collaborative groups work best if students are grouped according to like abilities.	3.85	3.0	0.85	3.1	<0.01
Inverse of Young children must master math facts before starting to solve problems.	3.26	2.89	0.37	1.38	0.09
Math is primarily about communication.	2.65	2.35	0.30	1.83	0.04
Learners generally have more flexible solution strategies than their teachers or parents expect.	2.7	2.25	0.45	2.65	<0.01
Teachers should intervene as little as possible when students are working on open-ended mathematics problems.	2.85	2.55	0.30	1.3	0.10
Inverse of Teachers should make sure that students know the correct procedure for solving a problem	3.8	3.3	0.50	2.24	<0.01
Inverse of Math is primarily about learning the procedures.	2.5	2.2	0.30	2.35	0.01
Inverse of The idea that students are responsible for their own learning does not work in practice.	2.4	2.1	0.30	1.83	0.04
Overall 13 Item Composite Score*	2.77	2.34	0.425	5.93	<0.0001

* A 13 item composite score is calculated for each participating teacher as the average of the teacher's responses to each of the 13 Beliefs items listed in this table.

Table 2 shows the overall pre-test and post-test mean Belief scores for the subset of 13 belief items that are aligned with the intervention. The scores are interpreted as 1 (strongly agree), 2 (agree), 3 (uncertain), 4 (disagree), and 5 (strongly disagree). A composite pre and post beliefs score was calculated for each participating teacher as the average of the teacher's responses to these 13 beliefs items. The overall composite pre-test and post-test Beliefs Inventory

scores, on the average, reveal a highly statistically significant growth in Beliefs of 0.425. This result may be interpreted as follows: The average pre-test Beliefs score of 2.77 revealed that for these 13 inventory items a participating teacher, on the average, scored approximately three-quarters of the way between 2 (agree) and 3 (uncertain). On the post-test, the participating teacher's average score shifted 0.425 units closer to 2 (agree). That is, the mean post-test Beliefs Inventory score shifted away from 3 (uncertain) to approximately 55% of the way toward 2 (agree).

Table 3 compares the Beliefs mean post-test growth for the 13 aligned Beliefs Assessment Inventory items and the 21 non-aligned Beliefs Assessment Inventory items. The 21 non-aligned items have a statistically non-significant post-test gain of 0.048 compared to a highly significant post-test gain of 0.425 for the 13 aligned inventory items

Table 3

Comparison of Composite Participating Teacher Belief Scores for Aligned and Non-Aligned Experimental Program Belief Assessment Inventory Items

Belief Assessment Composite Mean Score	Mean Pre-Test Score	Mean Post-Test Score	Mean Growth	Student Ratio	Significance Level
Average of 13 Aligned Belief Assessment Items	2.77	2.34	0.425	5.93	<0.0001
Average of 21 Non-Aligned Belief Assessment Items	2.18	2.13	0.048	0.85	N.S.

Figure 9 compares the participating teacher growth distribution of the aligned and non-aligned mean Beliefs Inventory items responses. It should be noted that for the aligned Beliefs Inventory items, on the average, less than 10% of the participating teachers did not exhibit any positive growth; in contrast, for the non-aligned Beliefs Inventory items over 50% (or precisely 52.6%) of the participating teachers failed to exhibit any positive growth with regard to the non-aligned Belief Inventory items.

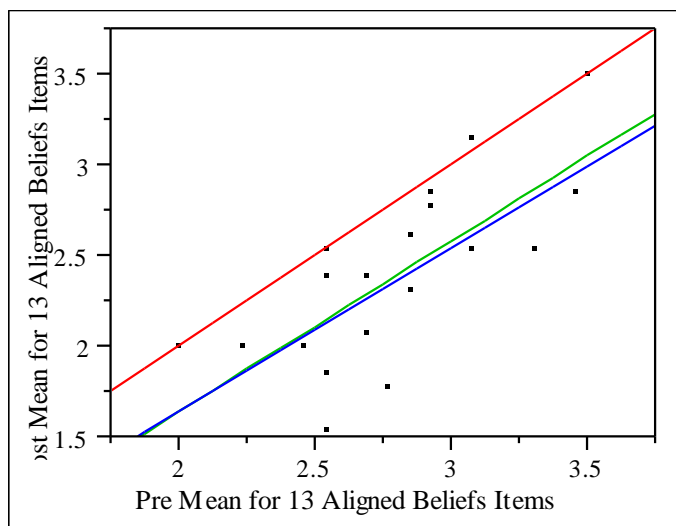Figure 10 is a graph of the mean 13-item aligned post Beliefs items

versus the corresponding pre Beliefs items for each of the 20 participating teachers. The graph contains separate linear regression plots for the participating regular and special education teachers. These graphs indicate similar growth in post Beliefs Inventory results for these two groups of teachers. The line y = x (or Post Beliefs Assessment mean score = Pre Beliefs Assessment mean score) is included in the graph to show the magnitude of Beliefs post-assessment growth as a function of the participating teacher's mean Beliefs pre- assessment score. Note that the vertical distance between the red line and the green line at any point on the pre-assessment axis is the corresponding expected mean post-assessment growth for regular classroom participating teachers. Similarly, the vertical distance between the red line and the blue line at any point on the pre-assessment axis is the corresponding expected mean post-assessment growth for special education classroom participating teachers.

Aligned Beliefs Assessment Item Composite Growth Scores	Non-Aligned Beliefs Assessment Items Composite Growth Scores
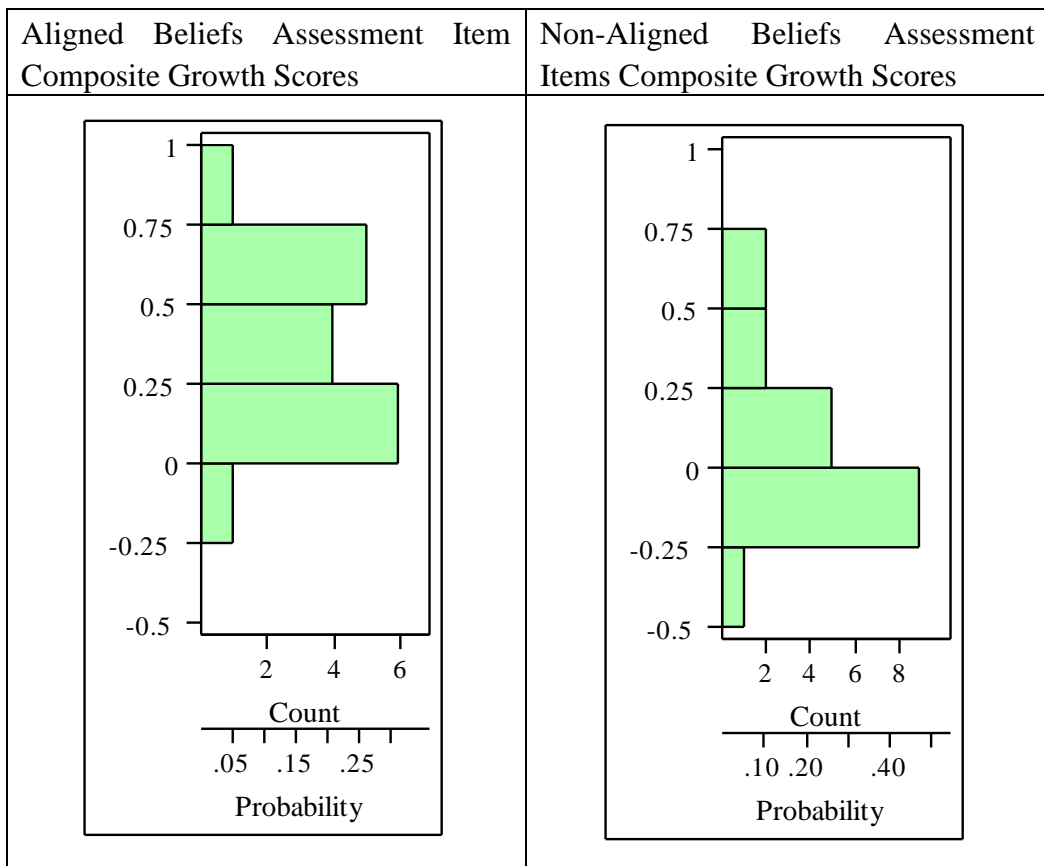
Quantiles				Quantiles		
100.0%	maximum	1.000		100.0%	maximum	0.5238
99.5%		1.000		99.5%		0.5238
97.5%		1.000		97.5%		0.5238
90.0%		1.000		90.0%		0.5238
75.0%	quartile	0.615		75.0%	quartile	0.2381
50.0%	median	0.462		50.0%	median	-0.0476
25.0%	quartile	0.154		25.0%	quartile	-0.1429
10.0%		0.000		10.0%		-0.2381
2.5%		-0.077		2.5%		-0.2857
0.5%		-0.077		0.5%		-0.2857
0.0%	minimum	-0.077		0.0%	minimum	-0.2857

Moments			Moments	
Mean	0.4251012		Mean	0.047619
Std Dev	0.3123246		Std Dev	0.2433287
Std Err Mean	0.0716522		Std Err Mean	0.0558234
upper 95% Mean	0.5756368		upper 95% Mean	0.1648997
lower 95% Mean	0.2745656		lower 95% Mean	-0.069662
N	19		N	19

Figure 9. **Distribution of participating teacher growth for aligned and non-aligned beliefs assessment item composite scores.**

> ——Expected Results if No Change in Teacher's Post – Test Scores
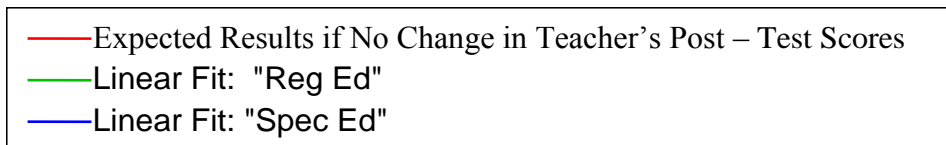> ——Linear Fit: "Reg Ed"
> ——Linear Fit: "Spec Ed"

Figure 10. **Participating regular classroom and special ed teachers - post vs pre Mean 13-Aligned Item Belief Inventory Scores.***

Discussion

Teachers were actively engaged in building justifications to problems, having available concrete objects (unifix™ cubes) to spontaneously construct representations of tower arrangements. In their activity, they discovered patterns and organizations that enabled them to classify towers according to some scheme. Their actions resulted in ways of organizing the towers. While initially not always producing complete organizations, teachers eventually came to build thoughtful and convincing arguments for their solutions. In their problem solving, they integrated the ideas of others, as was evidenced by their written work and explanations.

Their active engagement as mathematical learners was, in our view, a necessary, but not a sufficient prerequisite for understanding the reasoning of children. In order to understand students' ways of doing mathematics, the studying of videos of children's mathematical activity provides an entry and an enticement. The videos show problem-solving behavior not unlike that of the adult learners. They indicate the benefit of activity and reflection in children's learning. Given the opportunity for experimental teaching, by observing and working with children in their own classrooms and working with colleagues to analyze their students' reasoning, teachers can learn to become more observant of the mathematical behavior and forms of reasoning of children that naturally evolve in the process of problem solving. Steffe (2010) emphasizes the importance of teachers' learning to engage in children's productive mathematical thinking by engaging in teaching experiments and working as teacher-researchers. We suggest that this engagement played an important role in influencing the beliefs of a significant number of participating teachers.

Our intervention suggests that a study of carefully selected videos of children doing mathematics can be an effective medium for helping teachers become more aware of the untapped potential of children to build mathematical ideas and ways of reasoning. Our approach sought to provide teachers with videos as tools for becoming more attentive to the developing

ideas of children. Further, it challenged teachers to examine strongly held earlier beliefs about how children learn and reflect on their own beliefs and behavior as classroom teacher. The children in the videos were not told how to solve the problems they were invited to work on; nor were they shown how to reason. Yet, using each other as a resource and the tools provided, they were successful.

It is interesting to note is that belief changes were exhibited by both regular and special education teachers, and that no differences were found, suggesting that students identified as struggling also can be successful in building meaningful solutions to problems. Changes in beliefs about how mathematics is learned and how teachers can influence children's learning may be a prerequisite to making changes in instructional methods. However it is not sufficient.

Actual or perceived obstacles can impede changes in practice even for teachers who realize the benefits of alternative approaches. Change requires the support of school administrators, who are accountable for the improvement of test scores, and parents who need to understand the benefits to their own children (Mueller, Yankelewitz, & Maher, 2010). Earlier research has shown that interventions that seek to establish understanding of mathematical concepts and that focus on problem solving do not affect students' computational competence (Maher, 1991). Educating the stakeholders about the necessity for children to build a strong foundational understanding of mathematics throughout the grades in order to continue a successful later study of mathematics in high school and beyond is a significant challenge (Francisco & Maher, 2005; Maher, 2005).

There are clearly limitations to the study. More research is needed to understand the complex relationship between mathematical knowledge, student reasoning and the impact of video based interventions. Also, follow up study in classrooms for growth in children's reasoning is also worthy of study**.**

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