

Framing Mistakes: Elementary Teaching Practices and the STEM Trajectories of Minoritized Students

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This intrinsic case study examines how one elementary teacher's epistemological and positional framing of mistakes, as learning opportunities rather than deficiencies, shaped the long-term educational trajectories of students from racially minoritized backgrounds. Drawing on in-depth interviews with three college students, Mario, Liliana, and Violeta, who are now pursuing STEM degrees more than a decade after having the same teacher in 2nd, 3rd, and 4th grade, the study explores how early classroom experiences rooted in affirming and inquiry-based error framing supported their mathematical identity development. The findings illustrate a teacher's instructional moves described by her former students that framed mistakes as meaningful learning opportunities and positioned them as capable mathematical thinkers, moves they credit with shaping their confidence and persistence in STEM over time. These findings reveal that when mistakes are framed as valuable resources and students are positioned as capable of learning from them, it fosters enduring academic confidence. This research underscores the long-term impact of elementary teaching practices on the success of minoritized students in STEM and offers critical implications for teacher educators seeking to prepare teachers to cultivate equitable, identity-affirming learning environments.

Keywords: elementary mathematics education, framing errors, STEM trajectories, minoritized students

In mathematics classrooms, mistakes are often framed as signs of failure, deficiencies, or indicators of students' low ability, particularly for those from historically marginalized racial and social backgrounds (Adiredja, 2019). Teachers with deficit-based framings tend to focus on identifying students' errors, misconceptions, or knowledge gaps and treat them as learning deficiencies (Scheiner, 2023). This deficit-oriented framing of mistakes, coupled with the exclusionary positioning of minoritized students, aligns with dominant classroom practices that frequently position students of color as

passive recipients of fixed procedures and prepackaged knowledge (Louie, 2017). This positioning restricts their opportunities to engage in meaningful mathematical reasoning, exercise agency, and develop positive identities as capable thinkers and doers of mathematics (Adiredja & Louie, 2020; Boaler & Greeno, 2000; Cobb et al., 2009). Further, such deficit-framed environments tend to promote a narrow vision of mathematics, one where the ultimate goal is to quickly produce the correct answer by applying rigid procedures, disconnected from conceptual understanding (Boaler & Greeno, 2000). In doing so, they deny students—especially those from marginalized communities—the right to fully participate in mathematical reasoning, sense-making, and identity development (Boaler & Anderson, 2018).

However, when teachers frame mistakes as valuable learning opportunities, they can help students, particularly those who are often subject to those exclusionary practices because of their race, language, or social status, to (re)position themselves as mathematically competent (Louie et al., 2021). This (re)positioning is not merely about improving academic outcomes; it is fundamentally about supporting students in developing a sense of mathematical competence and belonging (Louie et al., 2021).

Thus, exploring how teachers frame errors and how students make sense of those framings more than a decade later offers critical insights into the development of mathematical identities and the cultivation of agency among historically marginalized students. This study examines the case of one elementary teacher, Ms. Duran, who consistently framed mistakes as learning opportunities and positioned her students as capable mathematical thinkers. Through retrospective interviews with three of her former students, now successful college students majoring in STEM fields, this study specifically explores how her epistemological and positional framings of errors during their early schooling years influenced three of her students' long-term academic identities. The research questions guiding this study were:

1. How did Ms. Duran's epistemological framing of mistakes influence her students' perceptions of errors as learning opportunities during their elementary math experiences?
2. In what ways did Ms. Duran's positional framing of students during error-related interactions support the development of students' mathematical agency and intellectual authority?

Literature Review

The Reproduction of Exclusion through the Deficit Framing of Errors

Dominant practices in U.S. mathematics classrooms continue to privilege speed and correctness, often valuing procedural fluency over deep conceptual understanding. This emphasis reinforces a narrow definition of mathematical ability, one in which a student's capacity to quickly arrive at the correct answer is often viewed by teachers as a sign of proficiency or of being

“good at math” (Boaler, 2016; Boaler et al., 2018; Goffney et al., 2018). In contrast, students who struggle to produce correct answers or who make mistakes are frequently labeled as “low” or “deficient” (Louie, 2017; Santagata, 2004). As a result, errors, especially those made by students from historically marginalized groups, are often interpreted through a deficit lens (Adiredja, 2019) and may become the obsessive focus of teacher attention (Louie et al., 2021). In doing so, teachers deny those students who often struggle to produce correct answers the right to fully participate in mathematical reasoning, sense-making, and identity development (Boaler & Anderson, 2018). Further, this dynamic not only limits these students’ access to meaningful mathematical engagement but also undermines the development of positive mathematical identities and long-term participation in STEM fields (McGee, 2016).

Further, when mistakes are interpreted through a deficit lens and treated as signs of inherent limitations in minoritized students’ learning, teachers reinforce instructional practices that privilege fixed procedures and prepackaged knowledge (Adiredja, 2019; Louie, 2017; Scheiner, 2023). This deficit-oriented framing of mistakes restricts students’ opportunities to engage in meaningful mathematical reasoning and to exercise agency in their learning (Adiredja & Louie, 2020; Boaler & Greeno, 2000; Cobb et al., 2009). Such environments also promote a narrow vision of mathematics, where the ultimate goal is to know how to apply rigid procedures, often disconnected from conceptual understanding (Dorner et al., 2025; Boaler & Greeno, 2000). Thus, it is crucial to engage students in pedagogical practices that disrupt deficit narratives, particularly those that frame mistakes as signs of failure (Scheiner, 2023), especially for students of color.

Challenging Deficit Views: Errors as Resources for Learning

Decades ago, Freire (2016) emphasized that a responsible educational perspective must nurture students’ curiosity and creativity, an endeavor that requires rethinking the role of mistakes in the learning process. Freire (2016) described a mistake as “an integral part, an important moment, in the process of knowledge, or, to be more precise, of the process of creation of knowledge... The educator must then be prepared to deal with mistakes in a positive, encouraging, and challenging way” (p. 39). This view is particularly relevant for children from racially minoritized groups, who are often subjected to deficit narratives that frame their mathematical errors as evidence of low ability (Adiredja, 2019).

Building on these pedagogical perspectives, mathematics education research has increasingly emphasized the constructive role that mistakes can play in student learning (Borasi, 1987; Santagata, 2004). A substantial body of work highlights the importance of engaging students in error analysis as a means of developing critical thinking and problem-solving skills (Borasi, 1987, 1994; Kramarski & Zoldan, 2008; Metcalfe et al., 2024; Mathan & Koedinger, 2005; Zhuang & Conner, 2024). Such approaches encourage students to reflect

on their reasoning processes and view mistakes not as failure, but as valuable learning opportunities (Alvidrez et al., 2024b).

Promoting this kind of learning requires teachers to adopt discourses and practices that position students as capable of analyzing their thinking, even in the context of error (Kazemi & Hintz, 2014; Alvidrez, 2019). This involves not only reframing mistakes as learning opportunities but also engaging students in mathematical activities that allow them to share, reflect on, and refine their reasoning, thereby enacting their mathematical competence (Gresalfi et al., 2009). Teachers can reinforce this stance by explicitly positioning students who have made errors as contributors to mathematical discourse, rather than viewing them as deficient, and by framing their mistakes as valuable insights to be shared rather than signs of failure to be hidden (Lampert, 1990).

The positive role of mistakes is therefore central to the present study, which focuses on children from racially minoritized groups, specifically students of Mexican heritage, who are often viewed through deficit lenses that question their ability to produce mathematically “correct” or normatively ideal responses (Adiredja & Louie, 2020). By examining instructional practices that frame errors as learning opportunities and position students as capable thinkers and doers of mathematics, this study contributes to broader efforts to challenge deficit-based thinking and to foster equitable, identity-affirming mathematics learning environments (Alvidrez et al., 2024a).

Theoretical Framework

This study is grounded in Vygotsky’s sociocultural theory of learning, which views learning as a socially and culturally mediated process in which learners construct knowledge through interaction with objects, events, and experiences. According to this perspective, learning is non-linear and involves revisiting familiar ideas and contexts to generate deeper or new understandings. Learners make meaningful connections among and between these seemingly disparate experiences. As Brown and colleagues (1989) describe, “a concept will continually evolve with each new occasion of use, because new situations, negotiations, and activities inevitably recast it in a new, more densely textured form. So, a concept . . . is always under construction” (p. 33).

Within this sociocultural framework, teachers have a critical responsibility to create opportunities that support students in developing and challenging their own understanding, including learning through mistakes. Given that these processes often involve making and learning from mistakes, they take on particular significance for students from historically marginalized racial and social identity groups, who are especially vulnerable to being labeled as “low” or “deficient” in math classrooms. As such, analyzing how teachers frame errors and position their students is crucial to understanding how students

of color can develop identities as capable mathematical learners (Alvidrez et al., 2024a)

In the context of mathematics education, teachers' framing of mistakes plays a critical role in this process. A teacher's epistemological framing (what they communicate about the role of mistakes in learning) and positional framing (what roles they assign to students when errors occur) shape how students see themselves and their capacity to engage meaningfully with mathematics (Greeno, 2009). When errors are treated as resources rather than deficiencies, students can be positioned as intellectually capable of identifying, analyzing, and learning from their own mistakes.

By drawing on the framework of epistemological and positional framing of mistakes (Alvidrez et al., 2024a) this research highlights how teachers' interpretations and responses to student errors can shape students' mathematical identities, agency, and long-term academic trajectories, particularly for those from racially minoritized backgrounds. This paper builds on previous research by utilizing the epistemological and positional framing of mistakes framework (Alvidrez et al., 2024) to examine Mrs. Duran's students' views and perspectives on her framing of errors for supporting their math learning and participation during 2nd, 3rd, and 4th grade. However, unlike studies that analyze teachers' framing of mistakes during classroom interactions (Adiredja & Louie, 2020; Louie et al., 2021; Scheiner, 2021, 2023), this study offers a retrospective perspective grounded in students' recollections of their elementary experiences with Mrs. Duran's framing of mistakes. Together, these frameworks highlight how teachers' interpretations and responses to student mistakes can influence students' mathematical identities, agency, and long-term educational trajectories, particularly for students from racially minoritized backgrounds.

Methods

This study is part of a larger research project focused on understanding how the elementary educational experiences of students from racially minoritized backgrounds influence their trajectories into higher education (Alvidrez et al., 2024b). The purpose of this intrinsic case study is to elucidate the experiences of young learners who were positioned as capable doers and thinkers by their elementary teacher, who framed mistakes as learning opportunities in mathematics. Intrinsic case studies are used to understand a particular, unique circumstance (Stake, 1995), in this case, the experiences of three students who are now highly successful in higher education STEM fields and who attribute their academic trajectories, at least in part, to their experiences in an elementary classroom with her teacher Mrs. Duran. These students were enrolled in that teacher's class for three consecutive years, approximately a decade ago.

Context and Participants

The three participants in this study, Mario, Liliana, and Violeta (pseudonyms), were recruited using snowball sampling techniques (Naderifar, 2017). The first participant, Mario, was recruited after he emailed one of my mentors, Dr. Puentes, who was also a former professor of Mrs. Duran, to thank her for her donations to Mrs. Duran's elementary classroom, which had inspired his interest in STEM, particularly in engineering. This message motivated Dr. Puentes to contact some of her mentees to conduct a research investigation to understand Mario's and his peers' pre-college experiences and the impact of their elementary education (2nd to 4th grades) on those experiences. Mario then recommended Liliana and Violeta as potential participants, given that they met the participant criteria relevant to this study's objectives. Participant criteria consisted of participants who had been in Mrs. Duran's class from the years 2011 to 2013.

Over ten years ago, these students attended an elementary school located in a low socioeconomic Mexican community. Now, they are enrolled at an R-1 Hispanic Serving Institution on the US-Mexico border in western Texas. All students are Mexican and first-generation college-going. Mario is majoring in mechanical engineering, Liliana in computer science, and Violeta in geology. All three students have successfully completed their first year of college. Moreover, Violeta and Liliana, who are engaged in undergraduate research, have submitted and have had proposals approved for various conference proceedings. At the moment of the third interview, Violeta was conducting research at the North Pole as part of her summer internship program, while Liliana was preparing a conference proposal based on her work as an undergraduate research assistant.

Data Sources

Audio-recorded interview data was the primary source of data (Creswell & Poth, 2016) and was collected using Seidman's (2013) technique of focused, in-depth interviews. In the first interview, participants constructed meaning from their experiences during 2nd, 3rd, and 4th grade with Ms. Duran, as she had looped with the participants. The second and third interviews focused on participants' experiences through high school and their first year as college students, respectively. In this study, I am focusing on data collected from the first interview.

Data Analysis

The first stage of data analysis involved identifying the units of analysis, as the students' interviews addressed their experiences in Mrs. Duran's classroom without a specific focus on mathematics. Therefore, for this intrinsic case study, the selected units of analysis centered on the narratives students identified as meaningful in relation to their mathematical learning experiences during their 2nd, 3rd, and 4th grades. I coded the interviews for patterns using an

open and focused coding approach (Emerson et al., 2011). Across all the interviews, a common and recurring code that emerged was *errors*¹

Once this code was identified, the second stage of analysis involved a close examination of teacher-student interactions and classroom activities in which students described moments involving mathematical errors. This method enabled me to analyze Mrs. Duran's discourse and pedagogical moves through students' descriptions of moments when errors emerged. I employed an inductive analytic process to identify and code episodes in which Mrs. Duran used errors as learning resources and emphasized students' ability to cope with their mistakes. Throughout this process, I engaged in an iterative dialogue between theory and data, using the epistemological and positional framing of mistakes framework (Alvidrez et al., 2024a) to guide my interpretation of the moments described by participants.

These moments served as rich sites for analysis, revealing how Mrs. Duran's framing of mistakes supported students in developing positive mathematical identities that they reported as still shaping their STEM trajectories over a decade later. The examples shared in this paper are representative of instructional practices students described as consistent and characteristic of Mrs. Duran's classroom. Codes were then grouped under three themes: using hands-on activities as an alternative to analyze mistakes, small group error discussions, and redirecting attention to students' ability to learn from mistakes. The findings that emerged from this recursive process of coding and analysis are presented in the next section.

Findings

By tracing students' retrospective reflections, this research highlights the transformative potential of framing mistakes as resources and positioning students as capable learners who can learn from their mistakes. Thus, this paper illustrates how one elementary teacher's efforts to frame mistakes as learning opportunities and to position students who erred as capable supported meaningful learning in the math classroom and helped students develop positive mathematical identities. It is important to note that these participants affirmed "[they] still apply the methods learned in Ms. Duran's classroom as a way to understand the curriculum in her college math and science courses, more than 10 years later." This enduring understanding of errors as a natural and valuable part of learning continues to shape the students' academic confidence and persistence in STEM fields.

Using Hands-On Activities to Scaffold Student's Analysis of Mistakes

¹ In this study, I use the terms *error* and *mistake* interchangeably to refer to students' incorrect mathematical reasoning or responses.

Fostering students' conceptual mathematical knowledge requires providing them with opportunities to examine their thinking and define their problem-solving strategies, whether they are incorrect or unclear. Mrs. Duran's instructional approaches supported her students in developing these skills in her second-grade classroom by framing students' mistakes as learning opportunities. Mrs. Duran built and honed her students' skills, aiming to support them in detecting, correcting, analyzing, and learning from their mistakes was a vivid memory for her three students.

Participants recognized and explained how Ms. Duran used different approaches to help them reflect on mistakes in ways that allowed them to make mathematical connections and identify themselves as capable learners and doers. For example, Violeta explained how Mrs. Duran created and provided them with manipulatives to help them visualize their errors and improve their problem-solving strategies, "We used manipulatives to model our thinking because Mrs. Duran used to say, 'from there you were able to see how and where you were wrong. That is a way to detect your mistake and learn more when you discover how that happened.'"

This example from Violeta's narrative illustrates how Mrs. Duran's instructional practices supported students' conceptual understanding by positioning mistakes as essential learning tools. Rather than treating errors as endpoints or signs of failure, Mrs. Duran framed them as starting points for deeper inquiry, encouraging students to examine their thinking and reflect on their problem-solving processes. Violeta described how the use of manipulatives provided her with tangible, visual tools that allowed her to identify where her reasoning had gone astray and to revise her approach accordingly once she "discovered how that happened."

Mrs. Duran's instructional move positioned Violeta not as a passive student awaiting correction from her teacher, but as a capable mathematical thinker equipped with the resources to make sense of her own mistakes. Mrs. Duran's language, as recalled by Violeta ("from there you were able to see how and where you were wrong"), explicitly placed interpretive authority in the hands of the student. This discursive move aligns with an epistemological framing of errors-as-resources, wherein mistakes are understood as central to the construction of knowledge.

Such positioning can foster students' identities as competent learners, supporting a shift from being perceived, or perceiving themselves, as individuals who simply "make mistakes" to those who can actively learn from them. The moment recalled by Violeta is especially significant because it represents a pedagogical strategy that resists deficit narratives frequently applied to racially minoritized students. Violeta's clear memory of this practice, years later, also suggests the lasting identity-shaping impact of this framing.

In another example, Liliana said that Mrs. Duran helped her understand the reasoning behind her mistakes by analyzing the math concepts and ideas involved in every problem using manipulatives or other hands-on activities.

About this, Liliana said, “Mrs. Duran supported us to think about errors as something useful. For example, she provided us with opportunities to use different hands-on activities and asked us to document which method we believed was most effective for understanding math.” This example from Liliana’s narrative underscores how, in Mrs. Duran’s classroom, errors were not treated as isolated missteps to be corrected by the teacher, but rather as opportunities to deepen conceptual understanding through inquiry-oriented tools and strategies.

Liliana describes how Mrs. Duran provided students with opportunities to make sense of their errors and, more broadly, to make sense of mathematics using hands-on activities. She also highlights how Mrs. Duran promoted student agency. By engaging students in reflecting on their thinking, determining which strategies worked best for them, and documenting their learning processes (especially after making mistakes), Mrs. Duran demonstrated an instructional approach that went beyond simply framing errors as learning resources or positioning students as capable. Her student-centered pedagogy actively disrupted deficit narratives about students of color by positioning them as autonomous, reflective, and competent mathematical thinkers. Furthermore, Liliana recalled not just the activity, but the sense of ownership and autonomy it fostered. Through these moves, Mrs. Duran cultivated an environment in which students were not only allowed, but expected, to learn from mistakes, an expectation that positioned them as legitimate participants in mathematical sense-making.

Facilitating Peer Discussions to Promote Error Reflection

Using peer error discussion during small group teaching was another move used by Ms. Duran to frame mistakes as learning opportunities and position his students as capable. Participants recalled and described how Mrs. Duran used these groups to engage students in discussions about their mistakes and misconceptions. For example, Mario shared how Ms. Duran taught in small groups, and then she asked students to evaluate their own learning approaches by trying to explain their problem-solving strategies to each other, whether they had correct or incorrect responses. Regarding Mrs. Duran’s instructional small group peer teaching approach, Mario stated, “I thought that was very interesting, like an interesting way of doing it. Everybody had space to explain their responses when she allowed us to work in small groups. So, we had space with our friends to discuss our wrong responses; the teacher was not in charge of the talk. Instead, we were able to decide what was wrong and why.”

Mario’s reflection illustrates how Mrs. Duran’s use of small-group error discussions created a learning environment in which students were positioned as active agents in the process of error analysis. By intentionally stepping back and allowing students to work collaboratively, Mrs. Duran disrupted traditional hierarchies in the classroom where the teacher is the sole authority on correctness. Instead, she repositioned students as having the intellectual

authority to interpret, evaluate, and make sense of their own and others' mathematical errors.

Mario emphasizes that "the teacher was not in charge of the talk," underscoring how students were not only allowed but expected to engage in collective meaning-making from their error analysis. Thus, Mario's mention of discussing "wrong responses" with peers in a supportive setting highlights an epistemological framing of mistakes as resources for learning. In this context, error becomes a shared object of inquiry rather than a mark of individual deficiency. Moreover, the group setting also appears to have offered a low-risk, high-engagement context in which students could exercise autonomy and learn from one another, further reinforcing the social and intellectual value of errors.

Similarly, Violeta also described how she enjoyed working with her friends in small groups when she made mistakes. As she explained,

You know, sometimes when you make mistakes, you can feel like you're alone in it. But in Mrs. Duran's classroom, if you struggled to understand why you didn't solve a problem correctly, you just needed to ask a couple of friends to help guide you in figuring out those reasons.

Violeta's reflection underscores the social and emotional dimensions of error in mathematics learning, particularly how Mrs. Duran's classroom practices disrupted the isolation and shame that often accompany mistakes. Her statement, "sometimes when you make mistakes, you can feel like you're alone in it," points to a common affective experience in math classrooms where errors are often treated as individual failings. In contrast, Liliana's description of Mrs. Duran's classroom reveals an environment where errors were normalized as shared learning opportunities, rather than private struggles.

Violeta's use of inclusive language, "you just needed to ask a couple of friends," suggests that this was a consistent and accessible classroom norm. The simplicity and confidence in her phrasing reflect an internalized belief that learning from mistakes was expected and supported. In this way, Mrs. Duran's classroom not only reframed the role of error in learning but also repositioned students as interdependent and capable participants in mathematical discourse.

Redirecting Attention to Student Ability to Learn from Mistakes

Participants recalled and described how Mrs. Duran supported all her students to learn from their mistakes. To understand how she framed mistakes in ways that promoted her students' mathematical sense-making, it is important to examine how Mrs. Duran redirects students' attention to their ability to learn from their mistakes. For example, in the following data excerpt, Liliana illustrated how Mrs. Duran positioned her as capable of identifying, analyzing, and correcting her mistake(s). Mrs. Duran positioned Liliana as a capable student by involving her in the process of handling her errors:

If we would get things wrong, [Mrs. Duran] never really shamed us into like, “Oh, but you’re wrong. And that was a dumb answer for saying that.” It was always “Okay. So, let’s figure out how you got to that answer. And let’s figure out why it’s not right and figure out where to go from there.” And that helped a lot. That helped me to understand that I could do math not only in her classroom, but now as a CS student.

This excerpt from Liliana’s narrative provides a compelling example of how Mrs. Duran actively redirected students’ attention from being wrong to understanding why an error occurred and what could be learned from it. Liliana recalls that Mrs. Duran never shamed students for incorrect answers but instead initiated a process of collaborative inquiry into the students’ reasoning. This instructional move aligns with epistemological framing of errors as resources. From a positional framing perspective, this moment is critical in how it positions Liliana: not as a passive receiver of correction, but as an active participant in diagnosing and resolving her mistake. Involving Liliana in this process signals that she has both the capacity and the responsibility to engage in mathematical reasoning and reflection. This framing authorizes her to take intellectual ownership of her learning.

The long-term impact of this positioning is evident in Liliana’s reflection: “That helped me to understand that I could do math not only in her classroom, but now as a CS student.” This statement illustrates the long-term impact on students’ identity initiated in Mrs. Duran’s classroom. Thus, this suggests that positioning students as capable of learning from their mistakes can have lasting effects on how they see themselves as learners, even beyond K–12 education and into STEM fields.

Discussion and Conclusion

This study contributes to a growing body of research that challenges deficit-based narratives in mathematics education by centering the pedagogical value of mistakes (Louie, 2017; Louie et al., 2021; Scheiner, 2023). Where dominant practices in U.S. classrooms continue to equate speed and accuracy with mathematical ability (Louie, 2020), this study offers a counterexample: a classroom environment in which conceptual understanding was cultivated through discussion, reflection, and shared analysis of mistakes. These practices created the conditions for students to develop positive mathematical identities early in their academic trajectories, identities that proved durable and empowering over time (Alvidrez et al., 2024b)

My findings align with previous research that illustrates how teachers productively enact epistemological framing of mistakes as resources and a positional framing of students as capable learners (Alvidrez et al., 2024a) This study also aligns with previous research demonstrating the positive impact of learning from mistakes under supportive conditions, such as when students

engage in self-exploration of their incorrect answers and participate in collective argumentation around their errors (Metcalf et al., 2024; Zhuang & Conner, 2024). More specifically, this intrinsic study identified pedagogical practices and a teacher's moves that encourage conceptual understanding, promoting agency, and supporting the development of *persisting* positive mathematical identities (Gresalfi et al., 2009; Kramarski & Zoldan, 2008; Mathan & Koedinger, 2005; Alvidrez et al., 2024a) of students who are often excluded from these opportunities because of their socio and racial backgrounds. However, what most distinguishes this study is its retrospective lens, showing how former students, now successful in STEM fields, continue to draw on the learning dispositions and problem-solving strategies they first encountered in a classroom where mistakes were normalized, shared, and deeply examined.

Coming from communities that have historically been marginalized and oppressed, the students in this study not only have excelled in science, technology, engineering, and math (STEM) fields, but they have challenged dominant narratives that predict lower academic success for Mexican heritage students (Alvidrez et al., 2024b). Thus, the significance of this study lies in its exploration of how an elementary teacher's epistemological framing of mistakes as learning opportunities and her positional framing of students as capable learners can meaningfully influence long-term academic trajectories, especially in STEM.

These findings underscore the importance of creating classroom environments where mistakes are welcomed as part of the learning process and where all students, particularly those from racially minoritized backgrounds, are positioned as capable of engaging in mathematical reasoning. For teacher educators, this research affirms the value of preparing future teachers to critically examine their own assumptions about student ability, to adopt asset-based pedagogies, and to develop instructional strategies that normalize errors and leverage them as tools for growth. Framing mistakes productively is not simply a matter of technique; it is a matter of justice and inclusion.

This study has several important implications for mathematics teacher education, classroom practice, and educational equity. First, it highlights the need for teacher preparation programs to explicitly address how teachers' epistemological and positional framings of mistakes shape students' learning experiences and long-term academic identities. Second, the findings point to the necessity of cultivating classroom cultures that position all students, particularly those from racially minoritized backgrounds, as capable of engaging in mathematical thinking. This involves not only shifting how teachers respond to mistakes but also how they design learning environments that foster peer collaboration, encourage inquiry, and make space for student voice in moments of uncertainty and error. Third, this research emphasizes the long-term impact of early mathematics instruction on students' persistence in STEM. Educational leaders and policymakers should invest in sustained

professional development that equips elementary teachers with tools to frame mistakes productively and to recognize the broader identity work embedded in classroom interactions.

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