Journal of Mathematics Education
2025, Vol. 18, No. 1, 1-10

https://doi.org/10.26711/007577152790180

Introduction to the Special Issue: Mathematics Teachers' Professional Knowledge and Competencies

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As members of the mathematics education community at large, we have the responsibility of preparing students for advanced studies in mathematics. Given that some students leave school with limited mathematical capabilities, in addition to an increasing emphasis on 21st-century skills (e.g. creativity, innovation, critical thinking, communication), a priority task for us all is supporting development of these capabilities in students. Relatedly, it has become more important to know what types of mathematical knowledge, understanding, and skills are most needed by teachers in their teaching. This is especially crucial for mathematics teacher educators in considering how best to structure the content of their delivery for future and/or practicing teachers to support them in the development of relevant knowledge, understanding, and skills. Along these lines, many researchers consider teachers' cognitive abilities and affective characteristics as fundamental aspects of teacher and teaching quality (e.g., see Blömeke & Delaney, 2012). This *Special Issue* in the *Journal of Mathematics Education* aims to highlight the progress in discussions on mathematics education research regarding to the theme: *Mathematics Teachers' Professional Knowledge and Competencies*. The goal of the *Special Issue* was to focus on a wide range of critical areas and contribute to the global discussions on the knowledge and competencies of mathematics teachers for teaching mathematics in K-12 settings and beyond. Areas of interest included essential professional knowledge and competencies for teaching mathematics; how these knowledge and competencies are conceptualised; how they are measured; and how they are connected to teaching practices and student mathematics learning. We invited contributions addressing a wide range of mathematical topics and research perspectives, covering various contexts, and encompassing different school sectors as well as higher education with a variety of research types, i.e., classroom-based research, projects, and theoretical perspectives.

Teacher Professional Knowledge and Competencies

For more than three decades, significant efforts have been made in the field of mathematics education to examine what knowledge mathematics teachers should have for the teaching of the subject, how this knowledge is acquired, and how it grows. Recent examples include Topic Study Groups at the International Congress on Mathematical Education (ICME) and the Congress of the European Society for Research in Mathematics Education (CERME). Other significant works include the chapters by Petrou and Goulding (2011) and Ruthven (2011) in Mathematics Knowledge in Teaching (Rowland & Ruthven, 2011). As part of these efforts, scholars in the fields of mathematics teacher education and development have suggested various frameworks and models of the teacher knowledge base to describe the notion of teachers' mathematical knowledge. These include Fennema and Franke's (1992) Model of Professional Knowledge, Ball et al.'s (2008) Mathematical Knowledge for Teaching (MKT), which stems from Shulman's (1986) Pedagogical Content Knowledge (PCK), and Rowland et al.'s (2005) Knowledge Quartet.

This *Editorial* does not aim to review the extensive body of work on teacher professional knowledge and competencies in mathematics. This area has been reviewed extensively, including a recent narrative review conducted by Depaepe et al. (2020). Interested readers are referred to these reviews. In the current *Special Issue*, teacher professional knowledge was broadly defined as the knowledge, skills, beliefs, and experiences effective for classroom practice and the propensity to apply these in different situations (Blömeke & Delaney, 2012; Kaiser et al., 2017).

Reflection on the Papers Appearing in This Special Issue

The *Special Issue* contains five contributions, and each makes a valuable contribution to mathematics education research on teacher professional knowledge and competencies. To begin, with the rise of integrated learning and STEM education, there have been debates about the place of mathematics topic-specific knowledge and understanding in these learning settings. Furthermore, in today's societies, students need to develop additional skills during their schooling, such as teamwork, creative and innovative thinking, and entrepreneurship. Building on this line of research, *Borden et al.* investigated teacher interactions within small groups of students when these students were engaging in an entrepreneurial design project, an experiential learning context. The authors used the Teacher Discourse Moves (TDMs; NC^{2}ML, 2024) as a conceptual and analytical framework to examine which TDMs (if any) were used by six participating teachers, drawn on their personal practical knowledge and expertise. These TDMs consist of nine moves grouped into three broader categories:

1. Different types of questioning (advancing student thinking and assessing student understanding;
2. Intentional inclusion of student ideas (inviting student participation, assigning competence, attributing student thinking, and orienting to other's work; and

3. Creation of space for students to process and think about ideas (waiting, encouraging in-progress thinking, and revoicing).

The deductive analysis of video-recorded interactions between the teachers and their students has revealed that all TDMs were observable except for waiting. The first category of TDMs was used by the teachers more often than the other two categories of TDMs. Specifically, advancing student thinking and assessing student understanding moves were observed the most, whereas encouraging in-progress thinking and attributing student thinking moves occurred the least. According to the authors, the teachers activated their personal practical knowledge largely "for these small group conversations consisted primarily of questions and comments that sought to understand what the students were doing and then move students towards the successful completion of the entrepreneurial design project" (p. 21). They recommended professional learning opportunities for teachers to enhance the least observed moves: encouraging in-progress thinking and attributing student thinking to support student mathematics learning and identities.

Gates and Earls were guided by the assumption that future and/or current mathematics teachers' professional knowledge and capabilities are influenced by those of their mathematics teacher educators (MTEs). They aimed to know what professional learning activities MTEs engage in during their learning trajectories, as evidenced through a review of the relevant research literature. They defined learning trajectories "to include the activities and actions undertaken by a mathematics teacher educator that will support their professional work in the arenas of teaching or research within their field" (p. 28). The overall criteria for the inclusion of papers in the review were that the research had concerned professional learning of university MTEs, that were published in a peer reviewed journals after 2018, that was empirical, and that included an orientation toward professional growth and a professional learning trajectory. The 24 identified papers were reviewed based on an analytical framework adapted from the literature concerning the interactions that occur in complex classroom instruction settings. According to the authors:

> In this [analytic] framework, the professional learning organizer represents the planner or facilitator of the professional learning activity, however, it does not indicate that the person is necessarily providing or sharing expertise. The learner or learners are participants in the learning activity who is attempting to grow their professional learning. The professional learning activity includes any experience that could credibly be assumed to lead to an increase in professional knowledge. These might include a range of actions like reading, conversing, observing, creating, and practicing in the professional context. (p. 33)

The review of these papers has identified four categories wherein the respective MTEs gained professional learning:

1. Solo professional learning (e.g. in supporting practicing teachers' professional development, in design-based action research, an MTE had gained understandings into practicing teachers' professional learning as well as how to integrate theoretical and practical knowledge).
2. Collaborative professional learning – collaboration throughfellowships or the same institutional affiliation (e.g. a group of MTEs from different institutions collaborated through fellowship opportunities and conducted a collaborative self-study; critical friendship provided professional learning for MTEs to support their preservice teachers in developing equitable practices in classrooms).
3. Collaborative professional learning – collaboration based on a common interest (e.g. a group of MTEs met in a working group at a conference and conducted a collaborative self-study on improving (self) questioning practices).
4. External professional learning – professional learning offered to MTEs by others (e.g. MTEs participate workshops, conferences, micro learning classes within their institutions or outside).

The review has shown that the number of papers in the last category was smallest (3 papers), followed by the first (5 papers) and the third (6 papers) categories. A total of 10 papers were classified into the second category, making it relatively popular. Synthesising the findings from these categories, the authors outlined several key findings. Firstly, nine out of the 24 studies included prospective or early-career MTEs, and in the remaining studies the career stages of participants were unknown. Secondly, the MTEs in the 24 studies that were categorised into (1), (2), and (3) had dual roles: they were both the organiser and the learner in the professional learning activity. Next, professional learning in these studies gained mostly in/from informal networks, making the role of memberships to these groups crucial and revealing that professional learning was gained outside of the MTEs' institutions. And finally, the content of the professional learning in these studies were predominantly around classroom practices or theory; only in three studies the focus of the professional learning was mathematics itself. Acknowledging the particular limitations of the review, the authors has provided thoughtful future research recommendations.

Considering noticing and questioning aspects of a teacher's professional capability, with respect to responsiveness to children's mathematical thinking, *Dunning and Jacobs* examined the growth in a teacher's (Ms. B's) attention to her students' thinking. This investigation was situated in a larger body of research on responsive teaching in elementary mathematics. Analyses of interviews and on-the-job observations revealed growth in Ms. B's professional practices, both as a classroom teacher and a mathematics coach, resulting from her participation in long-term professional learning sessions. The study identified three key contributions, which I cite using the authors' own words:

> First, we [the authors] provided illustrations of what centering children's thinking can look like when teachers are in the process of developing

> competency. For instance, Ms. B showed increasing curiosity about the details of children's thinking in their strategies as well as an increasing ability to make sense of them. She also showed increasing use of these details as a starting point for follow-up questions rather than formulating follow-up questions only to achieve correct answers or desired strategies. Second, we identified a pivotal AHA moment in Ms. B's growth storyline—appreciation of the need to shift her questioning goals from getting children to correct answers and advanced strategies to understanding children's existing thinking. This seemingly small shift was a catalyst for Ms. B's growth, and she regularly referenced this AHA moment during Years 2–3. Third, we identified several tensions teachers are likely to face when learning to center children's thinking. For instance, Ms. B experienced uncertainty and discomfort with what it meant to make sense of and center children's thinking. She also experienced conflict between her intentions to understand children's thinking and the pressure she felt to fix errors and move to advanced strategies. (p. 63)

The practical implications of these findings were presented. Among them, the authors highlight that attentiveness or responsiveness to children's mathematical thinking can create fertile professional learning opportunities for teachers, as every child can have their own unique and potentially creative way of thinking in mathematics.

Bardy et al. focused on a relatively less investigated but growing research area: broadly, the professional competencies of facilitators in mathematics, many of whom are still teachers of mathematics. Backing up with the relevant research literature, *Bardy et al.* assumed that facilitators who conduct professional development for mathematics teachers can be highly experienced classroom teachers, but facilitating professional development requires additional professional competencies. For instance, teaching mathematics would largely require practical knowledge or mathematics pedagogy, and while having practical knowledge could equally be needed for facilitators, they would also need theoretical knowledge which classroom teachers might not always have or need to have. Whilst there are significant research studies focusing on professional knowledge and skills of facilitators, according to the authors, much needs to be learned. They therefore aimed to examine to what extend mathematics professional development (PD) facilitators could differentiate between their professional skills as a classroom teacher and a facilitator. In order to achieve this goal, they used a Three-Tetrahedron Model adapted from a relevant study. In this Model, there are three levels, and at each level, there are associated elements that are supportive to learning. The first level is Classroom Level where learners are students, and the associated elements include classroom mathematical content and classroom resources. The next is Teacher PD Level, and at this level learners are teachers. The associated elements at this level are teacher PD content and teacher PD resources. The third level is Facilitator PD Level where learners are facilitators and the associated elements included facilitator PD content and facilitator PD resources.

Being grounded in a multi-level PD program, informed by the Three-Tetrahedron Model, the authors generated qualitative data from 30 novice facilitators. These data were analysed using an analytical framework in which there are four required aspects of expertise, at both Classroom and Teacher PD levels. Broadly, these aspects are:

1. Jobs (tasks of teaching that teachers conduct in a classroom when teaching or of facilitation that teacher facilitators use in a teacher PD setting as facilitators).
2. Pedagogical tools (artifacts and instruments that teachers/facilitators design for teaching/facilitating).
3. Mediums for thinking and perceiving (conceptual and non-prepositional knowledge that can influence teachers'/facilitators' thinking and perceiving in relevant practices).
4. Orientations (subject-related attitudes that can influence practices).

According to the authors, effective facilitators have expertise in these four aspects not only at Classroom Level but also at Teacher PD Level. Sophisticated data analyses revealed that the Three-Tetrahedron Model and analytical framework were effective in answering the study's research questions. The offered PD program helped participating novice facilitators become aware of the differences between the professional knowledge and skills they relied on in a classroom and in a teacher PD setting. Moreover, they were able to link their professional knowledge and skills they used in a classroom with those used in a teacher PD setting (bottom-up), or the ones that used in a teacher PD setting with those used in a classroom (top-down). As these links were made more frequently towards the end of the PD program, and as the novice facilitators had become to be more proficient in their facilitator roles, the study found suitable PD programs necessary for teacher PD facilitators.
Hatisaru et al. presented an examination of preservice secondary mathematics teachers' (PSMTs') strategic competence for solving algebraic word problems. The authors developed and applied a strategic competence assessment rubric that extends general problem-solving steps. This rubric provides a broader perspective for examining strategic competence of individuals, operationalised around six strategic competence indicators (SCIs):

SCI 1: Understand the problem and its key features
SCI 2: Formulate the problem mathematically
SCI 3: Know at least one sophisticated solution method
SCI 4: Arrive at a correct answer
SCI 5: Know a variety of valid solution strategies
SCI 6: Demonstrate competency in using an efficient procedure

The authors situated the current investigation in their larger qualitative research conducted with initial teacher education students at an Australian university, where PSMTs' content knowledge for teaching algebra was investigated. Here, they aimed to obtain an in-depth understanding of the strategic competence construct, in responses to an algebraic word problem including simultaneous equations (named as the Dice problem) given by 19 PSMTs. The participants were asked to solve the Dice problem in as many ways as they could. A total of 42 solution methods were identified in their responses. In-depth analyses of these methods, in light of the six SCIs, revealed both strengths and weaknesses in their strategic competence for solving the Dice problem. Specifically, most of the PSMTs showed reasonable strategic competence in response to solving the problem; the PSMTs who exhibited greater competence across the six indicators, generally presented elegant solutions. Also, most PSMTs were aware of alternative methods to the problem (algebraic and numerical). Algebraic methods mostly included using simultaneous equations; numerical methods were listing possible integer solutions to both equations and then looking for the common solution. While both algebraic and numerical methods used frequently (20 and 16 occurrences, respectively), there was an observable absence of graphical methods, with only 2 out of 42 total solutions employing graphical approaches. Based on their investigation and findings, the authors suggested that the strategic competence assessment rubric developed offers a tool that could be adapted for different mathematical problems and contexts in teacher education. It also has potential applications for both research and practice in mathematics teacher education.

Concluding Remarks

A small but selective number of papers published in this *Special Issue*, and the key concepts involved each are presented in Table 1.
Taken together, these five papers offer rich and connected views of teacher professional knowledge and competencies in mathematics education. *Hatisaru et al.* examine pre-service secondary mathematics teachers' strategic competence in solving algebraic word problems, while *Borden et al.* explore how teachers use discourse moves to support student thinking in entrepreneurial design projects. *Dunning and Jacobs* highlight the growth of teachers' responsiveness to children's mathematical thinking, and *Bardy et al.* investigate the professional competencies required for facilitators who lead teacher professional development. Complementing these, *Gates and Earls* provide insights into the professional learning trajectories of mathematics teacher educators, revealing the importance of informal networks and collaborative structures. What stands out across these contributions is the shared emphasis on growth: whether refining questioning practices, expanding solution strategies, or navigating dual roles, all underscore that professional knowledge is dynamic and deeply shaped by context and experience.

Table 1
Key Concepts Involved in Each Paper

Title	Keywords/concepts
Amplifying mathematics learning through teacher discourse moves: Investigating teachers' personal practical knowledge within an entrepreneurial design project (Borden et al.)	○ entrepreneurial design projects ○ mathematical discourse ○ personal practical knowledge ○ teacher discourse moves
Landscape review: State of professional learning for mathematics teacher educators (Gates & Earls)	○ mathematics teacher educators ○ professional learning ○ learning trajectories
A growth storyline for centring children's mathematical thinking: A grade 5 case (Dunning & Jacobs)	○ children's mathematical thinking ○ teacher learning ○ responsive teaching ○ noticing ○ questioning
Challenges of linking levels of classroom and professional development when qualifying facilitators (Bardy et al.)	○ novice facilitators ○ teacher professional development ○ facilitator preparation ○ three-tetrahedron model ○ expertise model
Strategic competence of pre-service mathematics teachers in solving algebraic word problems (Hatisaru et al.)	○ algebraic word problems ○ teacher knowledge ○ strategic competence ○ simultaneous equations

Acknowledgements

I wish to thank the authors who contributed to the current Special Issue and the reviewers who critically reviewed the submissions. I also thank Shuhua An, Editor-in-Chief for the Journal of Mathematics Education, for her support throughout the preparation of this issue, and Asli Koca-Ozgun for her help with the management side of the Issue.

Authorship Contribution Statement

This Editorial was conceived and prepared by the author as an individual contribution.

Declaration of Competing Interest

The author declare that she has no known competing interest that could have appeared to influence the work reported here.

Note: References marked with an asterisk (*) indicate papers published in this Special Issue.

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