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Strategic Competence of Pre-Service Mathematics Teachers in Solving Algebraic Word Problems

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Strategic competence is a key mathematical proficiency that teachers need in order to determine effective and efficient solution strategies for the problems that their students are likely to solve. This study investigated the strategic competence of secondary pre-service mathematics teachers (PMTs) based on their solutions to a word problem involving simultaneous equations. By considering six indicators of strategic competence: (1) understand the problem and its key features; (2) formulate the problem mathematically; (3) know at least one sophisticated solution method; (4) arrive at a correct answer; (5) know a variety of valid solution strategies; and (6) demonstrate competency in using an efficient procedure, the study revealed that some PMTs had a limited range of approaches to the problem, and in some cases, could not formulate an effective algebraic solution. The findings suggest that while PMTs may be equipped with certain procedural skills, there is a critical need for teacher education programs to enhance their ability to connect mathematical concepts across different representations and to foster a deeper understanding of algebraic thinking.

Keywords: algebraic problems, teacher knowledge, strategic competence, simultaneous equations

Strategic competence can be defined as one’s ability to adapt, formulate, represent, and solve mathematical problems in an efficient way (Kilpatrick et al., 2001). Thus, it can be considered as a measure of what individuals bring to a problem-solving situation. It is essential for teaching, not only because it implies mathematical skills, but also the ability to recognize alternative and efficient approaches. Researchers in mathematics education have adapted strategic competence in different ways in their studies. For example, Egodawatte and Stoilescu, (2015) defined it as “a logical series of actions, such as: analyzing information, constructing a problem representation, selecting tools for the solution, and planning the various steps to be carried out to reach the solution” (p. 292). Copur-Gencturk and Doleck (2021) defined it as consisting of three key indicators: devising a solution strategy, mathematising the problem, and arriving at the correct answer, in the context of multistep fraction word problems.

These ways of conceptualizing strategic competence represent models of problem solving in mathematics consisting of general heuristics. They have similarities to the well-established four-stage model of Polya (1945) consisting of understanding the problem, devising a plan, carrying out the plan, and looking back. While such conceptions of strategic competence may be "necessary to determine the direction of problem solving and to understand which stages to follow through in order to reach a solution" (Egodawatte & Stoilescu, 2015, p. 292), they do not take into consideration that it is intertwined with several other mathematical abilities, as implied by Kilpatrick et al. (2001). This suggests that it is important to consider other ways of conceptualizing strategic competence to include multiple interconnected indicators. Building on the work of Kilpatrick et al. (2001), Copur-Gencturk and Doleck (2021), we have developed and tested a model of strategic competence with indicators of evidence of it in one's mathematical work (Hatisaru et al., 2025a; Hatisaru et al., 2025b). This model consists of the following six strategic competence indicators (SCIs #):

SCI 1: Understand the problem and its key features.
SCI 2: Formulate the problem mathematically.
SCI 3: Know at least one sophisticated solution method.
SCI 4: Arrive at a correct answer.
SCI 5: Know a variety of valid solution strategies.
SCI 6: Demonstrate competency in using an efficient procedure.

In the current paper, we focus on these SCIs for secondary pre-service mathematics teachers (PMTs) in the context of solving algebraic word problems. The research question that guided our investigation is: What strategic competence is evident in PMTs' solutions to an algebraic word problem involving simultaneous linear equations?

Previous Research on Teacher Strategic Competence

Previous studies into teachers' knowledge have revealed gaps and a mix of influences affecting strategic competence, particularly in relation to worded problems where 'formulating' can be critical (e.g., Son & Crespo, 2009; Van Dooren et al., 2003). A number of studies have shown that PMTs may not have the expected mathematics problem solving skills (e.g., Barham, 2020; Yew & Zamri, 2016). Overall, the approach to addressing word problems was stereotypical and routine for PMTs who relied upon arithmetic approaches to complete these tasks, such as drawing pictures and using specific procedural approaches (Son & Crespo 2009). Yew and Zamri (2016) found similar results with eight secondary PMTs.

As noted in previous research, in some cases, arithmetic approaches were utilized rather than algebraic approaches, often depending on the teachers' teaching level focus, with secondary PMTs preferring algebraic approaches—sometimes at the expense of more efficient arithmetic approaches—and primary/elementary teachers favoring arithmetic (Van Dooren et al., 2003). Stronger problem solvers could adaptively apply varying approaches (algebra or arithmetic) based on the problem or context, suggesting a higher level of strategic competence. For teachers, the ability to identify a variety of solution strategies was often dependent on knowledge from previous content courses rather than what had been learned in teacher education courses (e.g., Cullen et al., 2017), although Barham (2020) suggested that development of skills is possible through problem-solving classes for PMTs. Barham (2020) also found that some PMTs relied on guess-and-check methods for solving algebra problems, no matter the grade level they were teaching.

It was evident in the studies that the varied strategies for solving algebra word problems can be related to PMTs' own content knowledge (e.g., Jorgensen & Lowrie, 2017) and strategic competence or lack thereof. More insight needs to be gathered on the strategic competence that PMTs possess, so that this area can be developed in teachers in order that they, in turn, can better support students.

Methods

The current is a qualitative study wherein it was aimed to obtain an in-depth appreciation of a construct of interest (strategic competence) in responses to an algebraic word problem given by a sample of secondary PMTs. The study was grounded in larger research conducted with initial teacher education students at a metropolitan university in Australia, where secondary PMTs' content knowledge for teaching algebra was investigated.

Participants

The total of 23 students (9 female and 14 male) who undertook the two mathematics pedagogy units offered in Semester 2 of 2022 were invited to participate. In these units, students develop pedagogical content knowledge for teaching Years 7 to 12 using the Australian curriculum, with activities based around mathematics content and problem solving. When the focus was on algebra, students worked on five algebraic problems (one problem per week) and completed a reflection form, as an opening activity for the class (15-20 minutes). Students recorded their solutions to the problems in the reflection forms in response to the prompt question: "Think of and explain as many different possible solutions to the problem as you can. Name the solutions as Solution A, Solution B, Solution C and so on".

Participation in the research was voluntary, and informed consent was obtained. A total of 19 students' solutions to one of the problems (Dice; introduced next) provide data for this paper. The participating students were named P1, P2, and so on, to ensure anonymity. Based on the course pre-requisites, the students likely had satisfactory performance in algebra and calculus-based mathematics topics at the end of secondary school; some had also completed tertiary mathematics units. Although not yet qualified as teachers, some had engaged in teaching practicum in schools, or worked informally tutoring high school students.

Data Source and Analysis

The Dice problem, presented in Figure 1, is a typical algebra word problem based on algebraic simultaneity that might be used in high school. Possible solution approaches include algebraic (elimination or substitution), numerical (taking the common solution of conditions *i* and *ii*, or using one condition to restrict the solution space considered for the second), and graphical (see Figure 1).

The 19 participants identified 42 solutions to Dice. These solutions might have been correct/incorrect, complete/incomplete, implemented, or suggested; all were counted. The scheme presented in Table 1 was utilized in data analysis to understand the strategic competence manifested in these solutions. The scheme was used to code the criteria using the scale: 'weak', 'moderate', or 'relatively strong' to explore the extent to which each SCI criterion seemed to be evident in the solutions (see Hatisaru et al., 2025b) for more details). For SCIs #3 and SCI #5, the solution strategies presented in Figure 1 informed the analysis. The solutions were examined and coded independently by Hatisaru and Richardson, with initial disparities discussed to reach consensus. The coding was then independently checked by Chick.

Figure 1

The Dice Problem and Example Solutions

Die A and Die B have twelve sides each. Suppose that you roll die A and die B at the same time. When do the dice satisfy the following two conditions?
(i) The sum of 2 times A plus B equals 15.
(ii) 3 times A minus B equals 5.
[Source: Ito-Hino (1995), cited in Hatisaru et al., 2025b)]

Taking the common part of conditions i and ii

(i) $2A + B = 15$

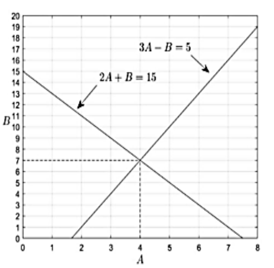
A	2	3	4	5	6	7
B	11	9	7	5	3	1

(ii) $3A - B = 5$ or $B = 3A - 5$

A	1	2	3	4	5	6
B	-2	1	4	7	10	13

The case that appears in both tables is $A = 4$, $B = 7$. Therefore, $(A, B) = (4, 7)$ is the answer.

Graphical



The intersection point of the two lines is $(A, B) = (4, 7)$.

The elimination method

(i) $2A + B = 15$
(ii) $3A - B = 5$

(i)+(ii) gives $5A + 0 = 20 \Rightarrow A = 4$

As $2A + B = 15$, $8 + B = 15 \Rightarrow B = 7$

The substitution method

$3A - B = 5 \Rightarrow B = 3A - 5$

$2A + B = 15 \Rightarrow 2A + 3A - 5 = 15$

$5A = 20 \Rightarrow A = 4$

As $B = 3A - 5$, $B = 12 - 5 \Rightarrow B = 7$

Results

Collectively, the participants performed well, with only 3 of the 19 failing to obtain correct values for *A* and *B* (SCI 4: Arriving at a correct answer, see Table 1). Table 2 shows the frequency of each of the remaining five SCIs for the participants. Findings regarding each SCI in the participants' solutions to the Dice problem are presented in the next subsections.

Table 1
Obtaining the Correct Answer by Participants

	Correct	Incorrect	Total
Participants	16	3	19

SCI 1: Understand the Problem and Its Key Features

SCI 1 was assessed based on whether the key mathematical idea in the problem is recognized. Most participants seemed to understand the idea of simultaneously being involved in the problem. Only three participants did not interpret the problem as meant that both conditions must be satisfied simultaneously, even though some could generate correct equations. So, their responses for this indicator were considered 'weak'. The remaining participants' responses for this indicator were 'relatively strong' (n = 16). Six recognized the simultaneous equations, while the remaining ten additionally implemented a second method where they showed that they needed to identify the solution that satisfied the two conditions.

SCI 2: Formulate the Problem Mathematically

This indicator was assessed based on whether the conditions *i* and *ii* were considered simultaneously (no matter which solution strategy is used), and whether they were translated into the relevant equations. Based on this assessment, only three of the participants' formulations of the problem were coded as 'weak – the same three coded 'weak' on SCI 1. Two approached the problem by tabulating values and algebraically, but their formulation did not allow them to reach a satisfactory solution. The third did not formulate the problem algebraically, using a numerical approach to successfully list solutions to condition *i* but without seeking a solution that also satisfied condition *ii*.

The remaining 16 participants' solutions were classified as 'relatively strong' as they considered conditions *i* and *ii* simultaneously and translated them into the correct equations. Among this group, P1 was able to formulate the correct equations and implement both standard algebraic techniques (elimination and substitution) for solving simultaneous equations (Figure 2, left). Their working out was reasonably clear, though they did not demonstrate explicitly how they obtained the value of *B* in Solution B, presumably deriving it mentally. For Solution C, it is unknown whether P1 knew how to identify the combinations of (*A, B*) values that satisfied condition *i*, or if they had a plan to then impose condition *ii*. P4 used a numerical approach as their primary solution method (Figure 2, right). Their second approach was algebraic, using the elimination method, although the working out did not describe the steps for solving for *B*.

Table 2
SCIs and Their Descriptions, with the Participants' (n = 19) Performance

Strategic competence indicator (SCI)	Description	W*	M*	RS*
SCI 1: Understand the problem and its key features	The idea of simultaneity was recognized.	3	-	16
SCI 2: Formulate the problem	When the problem is solved algebraically, the conditions *i* and *ii* were considered simultaneously, and they were translated into the equations $2A + B = 15$ and $3A – B = 5$. When the problem is solved by using other methods (e.g., numerically), it is formulated so that it can be solved by using mathematics.	3	-	16
SCI 3: Know a sophisticated solution method/s	Algebraic, graphical, and systematic numerical approaches are sophisticated solutions to Dice as they do not rely on random or unsophisticated guessing.	3	-	16
SCI 4: Arrive at a correct answer	The correct answer for the problem is given.	NA*	NA	NA
SCI 5: Provide a variety of valid solution strategies	Having at most one numerical *or* algebraic approach was considered as 'weak'; having at least one numerical *and* one algebraic solution approach was considered as 'moderate': having a third approach in addition to one numerical and one algebraic approach (e.g., graphical) was considered 'relatively strong'.	6	12	1
SCI 6: Demonstrate competency in using an efficient procedure	Using simultaneous equations is the most efficient approach for Dice, and that elimination is more efficient than substitution or isolation—although elimination does require the identification of an appropriate linear combination of equations, and substitution is more generalizable to non-linear systems of equations.	2	5	10

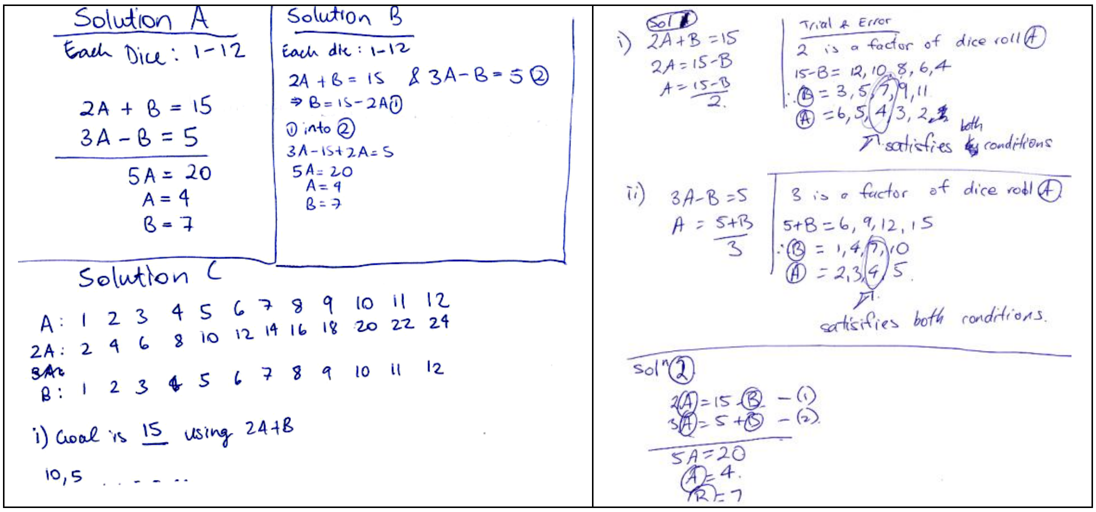
* W=Weak, M=Moderate, RS=Relatively Strong, NA=Not Applicable as it was coded as 'yes' or 'no'.

SCI 3: Know at Least One Sophisticated Solution Method

The Dice problem can be solved numerically, algebraically, graphically, or by matrix methods. As for SCI 3, an avoidance of exhaustive search or *random* guess-and-check methods is desirable. The three participants whose approaches to the problem were 'weak' used methods that would not produce the desired conclusion. For example, P6 formulated the two conditions algebraically, but did not suggest an algebraic or graphical method to solve them (Figure 3, left) with “Sol[ution] A” not clearly related to the problem nor showing how to solve the problem. “Sol[ution] B” was on the right path, with possible solutions to each individual condition listed, but P6 did not recognize the need to identify the solution that satisfied both. Concerningly, the reflective comments of all these three participants indicated that all believed that they had understood the question and presented a valid method. The other 16 participants used algebraic methods to solve the problem and laid their working out clearly. Many also implemented a second method which was clearly explained (see P13’s response in Figure 3, right). These participants’ strategic competence for this indicator was considered as ‘relatively strong’.

Figure 2

Responses of P1 (Left) and P4 (Right)



SCI 5: Know a Variety of Valid Solution Strategies

Regarding SCI 5, we looked at whether the problem was solved by using at least three different methods (e.g., numerical, algebraic, and graphical). An overview of identified strategies in participant responses is presented in Table 3.

The strategic competence of 6 participants was ‘weak’, meaning these participants were unable to identify—implement or suggest—more than one valid solution to the problem. Twelve participants provided at least one algebraic and one numerical strategy, classifying them as ‘moderate’. Among them, P1 and P9 solved the problem using both the elimination and substitution methods as their primary methods; P3 implemented the elimination method and suggested the substitution method as an alternative; P3 and P9 listed guess-and-check and table methods as alternatives, with limited indication of implementation. P1 also generated a numerical solution (in addition to solving the problem by using the elimination and substitution methods), although it was incomplete; and P22 solved the problem algebraically and suggested an alternative trial-and-error method.

Among all, only P14's response for this indicator was 'relatively strong' because P14 was the only one to suggest a graphical approach, in addition to implementing algebraic and numerical approaches. Although it was not actually carried out, they clearly described the method.

Figure 3

Responses of P6 (Left) and P13 (Right) to the Problem

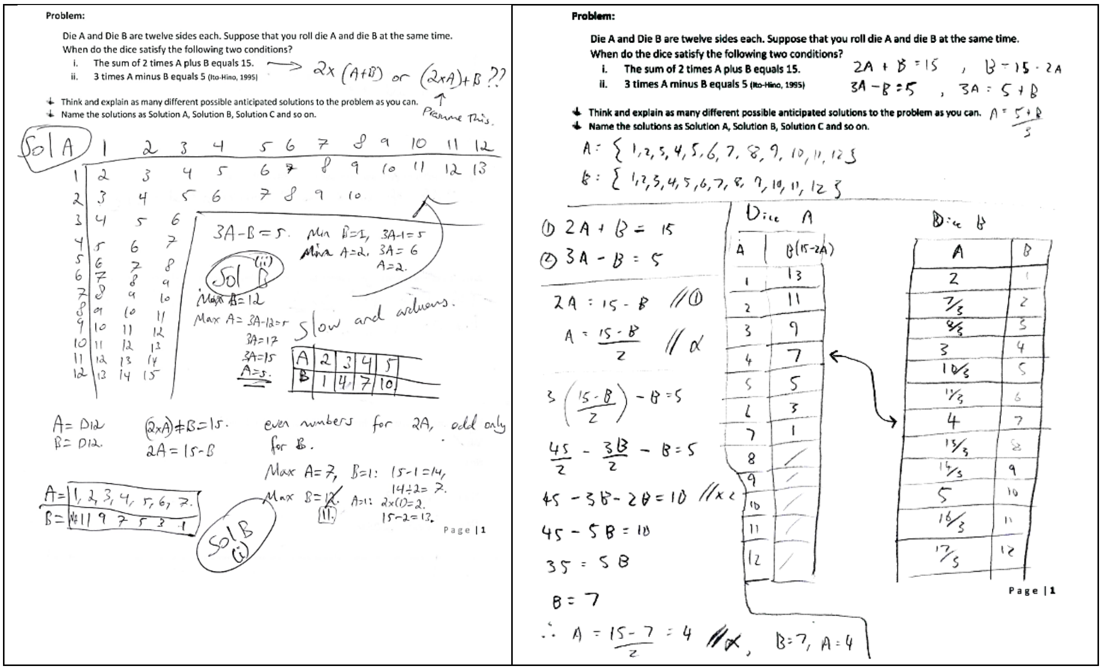


Table 3

Overview of Identified Strategies in Participant Responses

Algebraic	Numerical
Elimination (10)	List all solutions to *i* and *ii* and then find those common to *i* and *ii* (11*)
Substitution (9)	List all solutions satisfying condition *i* then check these in *ii* (1)
Ratio (1)	Guess and check (3)
Graphical (2)	Two-way table (1)

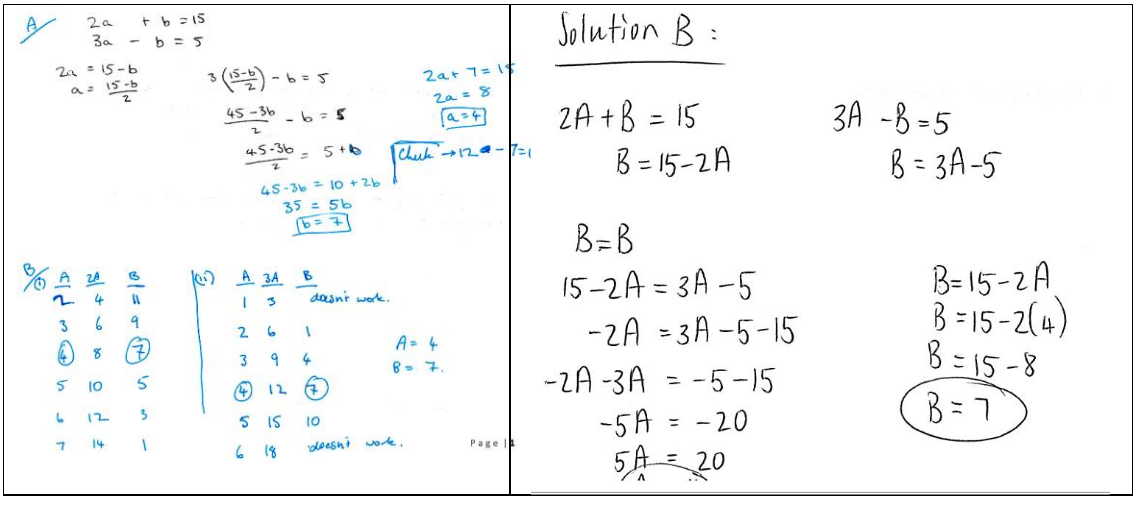
*1 of these was incomplete

SCI 6: Demonstrate competency in using an efficient procedure

As regards SCI #6, the use of simultaneous equations was deemed to be an efficient procedure for Dice, with a solution using elimination, or both elimination and substitution, coded as 'relatively strong', and other successful simultaneous equation methods deemed 'moderate'. Competency in using an efficient procedure could not be judged in two cases as P5 did not use an algebraic solution, and P7 wrote equations but did not recognize the need to solve them simultaneously. Here, it is noteworthy that classification as 'weak', 'moderate', or 'relatively strong' was conditional upon attempting to solve the problem algebraically.

In two cases, this competency was 'weak'. Specifically, P6 set up equations and manipulated them unsuccessfully (Figure 3, left), and P15 ultimately implemented the substitution method successfully; however, P15's working was chaotic. Of the responses classified as 'moderate', four—including P17 (Figure 4, left)—implemented the substitution with clear working out, although these participants did not make the simplest substitution choice. That is, they substituted an expression for *B* and then solved for *A*, which meant they had to deal with fractions. P20 used an unexpected method with clear working out: the isolation method (Figure 4, right), isolating *B* for both equations and then equating the resulting expressions involving *A*. Although these methods reflect competence with solving simultaneous equations, it is unknown if these participants were aware of the relatively simpler elimination method.

Figure 4
Responses of P17 (Left) and P20 (Right) to the Problem



Finally, the ten participants' competencies in solving equations were 'relatively strong' as they implemented the elimination method ($n = 6$), or both the elimination and substitution methods ($n = 2$, e.g., P1's solutions in Figure 2), or they implemented elimination and suggested substitution as an alternative method ($n = 2$).

Discussion

The analysis of secondary PMTs' solutions to the Dice problem revealed both strengths and weaknesses in their strategic competence. On the positive side, most of the PMTs showed reasonable strategic competence in response to the given task, suggesting that they are well-placed to demonstrate efficient approaches to their own students, and are aware of alternative approaches. The PMTs who exhibited greater competence across the six indicators, in Table 2, generally presented elegant solutions. More algebraic solution strategies (20 occurrences) were used than numerical approaches (16 occurrences) (see Table 3), with most of the latter involving listing possible integer solutions to both equations and then looking for the common solution. These results may align with previous research, where some were fostered due to the PMTs currently or recently engaged in the study of mathematics content (Barham, 2020; Van Dooren et al., 2003), noting that some participants might have had a more limited mathematics background prior to their teaching degree (Cullen et al., 2017).

There was a notable absence of graphical methods, with only 2 out of 42 total solutions employing graphical approaches (Table 3). This suggests PMTs may have limited understanding for the associations between algebraic and graphical representations, which is crucial for developing students' conceptual understanding of algebra. As emphasized by McCallum et al. (2010), establishing links between different representations is fundamental to mastery of algebra. This echoes Daniel's (2015) observation of PMTs' lack of relational understanding and conceptual knowledge in algebra. Such limitations in strategic competence can significantly impact PMTs' ability to provide conceptually rich instruction and support their future students' deep understanding of algebraic concepts. This disparity suggests that while PMTs may be comfortable with standard algebraic procedures, they may struggle with more abstract or general applications of these procedures. This finding aligns with previous Australian research indicating that while teachers may possess procedural knowledge, they may lack the deeper conceptual understanding necessary for more complex problem-solving (Daniel, 2015; Hatisaru et al., 2022; Wilkie, 2014). This gap in understanding has important implications for teacher education programs, which must ensure that PMTs not only learn how to perform algebraic procedures but also understand the underlying concepts that make these procedures work.

One noticeable feature of the collection of solutions presented by the PMTs was the informal approach to presenting work. Admittedly, they were not asked to produce display solutions, or solutions that might be suitable for sharing with students, and their responses might be deemed rough working out rather than a formal polished solution. Nevertheless, it is interesting to speculate the extent to which care with formal documentation might actually enhance strategic competence. On that basis, there is scope for providing experiences that strengthen strategic competence and further develop use of effective communication conventions for mathematical work.

In light of the fact that some PMTs demonstrated low performance on strategic competence indicators in solving the Dice problem, a potential direction of future investigation could be to examine word problems involving linear equations that can be solved using various methods—such as a table of values (numerical), forming and solving an equation (algebraic), or drawing a graph (graphical)—and compare the outcomes with those of the Dice problem. Specifically, using a word problem that requires less advanced versions of the solution methods may provide further insight into the strategic competence of PMTs.

Conclusion

In conclusion, by shedding light on the specific areas where PMTs excel and where they struggle, this research provides an understanding of the challenges might be faced by Australian future mathematics teachers, like also reported in Jorgensen and Lowrie (2017). The findings suggest that while PMTs may be equipped with certain procedural skills, there is a critical need for teacher education programs to enhance their ability to connect mathematical concepts across different representations and to foster a deeper understanding of algebraic thinking. At a methodological level, the strategic competence scheme used in this study can be adapted to different mathematics problems. This scheme can be used by teachers in schools and mathematics education lecturers at universities in their teaching methods courses to uncover their students' strategic competence.

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Authors' contribution statement
Investigation: VH and SR. Conceptualization and Methods: VH. Literature review: JS and VH. Data analysis: VH, SR, and HC. Findings: VH and SR. Discussion and Conclusions: HC, VH, and SR. All authors read and approved the final manuscript.

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