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Amplifying Mathematics Learning through Teacher Discourse Moves: Investigating Teachers' Personal Practical Knowledge within an Entrepreneurial Design Projects

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This study investigates how teachers facilitate small-group mathematical discourse during Design & Pitch (D&P) Challenges in STEM, an experiential framework combining entrepreneurial, project-based, and design-based learning. By examining teacher interactions, we explore how educators draw on their personal practical knowledge to support student thinking and navigate the unpredictable nature of student-generated ideas in these innovative learning environments. This exploratory study investigates how teachers drew on their personal practical knowledge during group check-ins, leveraging teacher discourse moves (TDMs) to facilitate student thinking and mathematics learning. We analyzed 14 video-recorded small-group check-ins, memoed for evidence of personal practical knowledge and coded using a priori TDMs drawn from the North Carolina Collaborative for Mathematics Learning (NC^{2}ML) framework. A thematic analysis of the coded transcripts revealed patterns in how teachers facilitated discourse, leading to three emergent themes: positioning students as experts, co-designing with students, and pushing students toward an outcome. These findings provide insight into how discourse moves can be used to support small group discussions within an entrepreneurial design project, as well as the challenges that emerge for teachers when guiding students towards desired outcomes.

Keywords: entrepreneurial design projects, mathematical discourse, personal practical knowledge, teacher discourse moves

Students in the 21st century must navigate pressing real-world issues and rapidly changing career opportunities, much of which humans are addressing with innovative solutions and an entrepreneurial spirit (Organization for Economic Cooperation and Development [OECD], 2018). These demands

call for learning experiences that cultivate skills such as critical thinking, problem-solving, communication, and collaboration—skills that real-world, project-based mathematics can effectively support (Abe, 2018). Entrepreneurship and design have also been shown to enhance student engagement (e.g., Lackéus, 2015; Mehalik et al., 2008), although they have rarely been used in mathematics learning environments.

This paper seeks to broaden the field's understanding of how teachers facilitate small group discourse within the Design & Pitch (D&P) Challenges in STEM (Confrey et al., 2019; Krupa et al., 2021), an experiential learning framework that leverages entrepreneurial-based, project-based, and design-based learning in mathematics classrooms (Belcher et al., 2025). One way to better understand how teachers support mathematical discourse in unstructured small group conversations is by observing teachers as they engage with students. This paper investigates teacher interactions within small groups of students as they engage in an entrepreneurial design project. Teachers were observed drawing on their personal practical knowledge (Connelly & Clandinin, 1988), leveraging mathematical discourse moves during small-group interactions to support students as they work through a D&P Challenge.

Literature Review

Experiential learning spaces like the D&P Challenges in STEM are designed to provide students with agency as they engage in learning experiences that reflect real world situations (Slavich & Zimbardo, 2012). A mathematics teacher's role in these learning environments is to elicit mathematical reasoning from students' ideas, which are typically unique and based on student interests (Belcher et al., 2025). In traditional mathematics classrooms, teachers have been observed drawing out their students' mathematical ideas by orchestrating mathematical discourse between their students (Herbel-Eisenmann et al., 2013). They often facilitate these discussions by employing specific teacher discourse moves (TDMs; Herbel-Eisenmann et al., 2013; North Carolina Collaborative of Mathematics Learning [NC^{2}ML], 2024), which are detailed in the analytic framework section of the methods. With the potential for widely varying student ideas in this entrepreneurial context (Belcher et al., 2025), a teacher must be able to think flexibly and draw on their personal practical knowledge (Connelly & Clandinin, 1988) to connect their ideas to the targeted mathematics as they respond to students during check-ins (Grossman et al., 2019). While little is known about the role of personal practical knowledge in facilitating mathematical discourse within entrepreneurial design projects, the demonstrated ability of TDMs to empower students in mathematics classrooms (Herbel-Eisenmann et al., 2013) suggests that these moves could be valuable tools in such settings. This literature review discusses the complexities of supporting mathematics learning within entrepreneurial design projects, how TDMs can be used to foster collaborative discourse in small groups, and how

teachers' personal practical knowledge (Connelly & Clandinin, 1988) manifests when facilitating student learning in the moment.

Mathematical Learning through Entrepreneurial Design Projects

Utilizing entrepreneurship in learning creates a space where "students can become highly motivated and engaged by creating value to other people based on the knowledge they acquire, and this can fuel deep learning and illustrate the practical relevancy of the knowledge in question" (Lackéus, 2015, p. 3). The D&P Challenges in STEM (Confrey et al., 2019; Krupa et al., 2021) were developed to engage students in mathematics through a project (i.e., project-based learning; Capraro & Slough, 2013) where they are positioned as designers (i.e., design-based learning; Mehalik et al., 2008) and entrepreneurs (i.e., entrepreneurial-based learning; Lackéus, 2015) that seek solutions to real-world challenges (Belcher et al., 2025). Throughout the project, teachers support students to iteratively reflect on and deepen their understanding of both the intended mathematics and the real-world problem they are solving (Confrey & Maloney, 2006).

Since entrepreneurship requires students to create unique ideas that fill an unaddressed corner of the market (Lackéus, 2015), the contexts in which they engage with mathematics are necessarily novel and hard to predict. Empirical evidence of this phenomenon was seen in a study on one of the D&P Challenges, where students engaged in the targeted mathematics in unanticipated ways that "were closely tied to students' interests in their chosen contexts" (Belcher et al., 2024, p. 22). Anticipating student solutions—an important first step in orchestrating productive mathematical discussions (Smith & Stein, 2011)—can therefore be especially difficult for teachers in these settings. To support mathematics learning in such entrepreneurial environments, teachers often need to check in frequently with student groups, monitor progress, pose probing questions, and facilitate engagement with key concepts (Belcher et al., 2025; Grossman et al., 2019).

Supporting Mathematical Discourse

In open-ended collaborative projects, students must collectively negotiate solutions (Capraro & Slough, 2013). Teachers play a critical role in facilitating effective discourse as students engage in these negotiations, particularly in an effort to ensure mathematical learning occurs (Herbel-Eisenmann et al., 2013; NC^{2}ML, 2024). Facilitated ineffectively, mathematical discourse can become unproductive, with students feeling unsupported or relying on authority figures (e.g., teachers) to lead the discussion and evaluate the correctness of responses (Esmonde & Langer-Osuna, 2013). Effective facilitation, however, creates a culture of respect and safety, where students feel encouraged to share their ideas (Herbel-Eisenmann et al., 2013; NC^{2}ML, 2024). When supported to justify their ideas and question and build upon the ideas of others, students can become more productive in their collaboration, even when

the teacher is no longer actively facilitating the discussion (Langer-Osuna, 2017). TDMs offer actionable strategies for teachers to promote student discourse and foster an environment in which all students feel empowered to contribute mathematical ideas (Herbel-Eisenmann et al., 2013; NC^{2}ML, 2024).

Teacher Expertise

Facilitating mathematics learning when students have the autonomy to pursue unique and unexpected solution paths requires teachers to improvise and respond to students in the moment. Teachers possess "personal practical knowledge" (Connelly & Clandinin, 1988, p. 25), developed through their experience and education, that can be beneficial in these moments. In practice, teachers regularly draw upon their previous experiences (Chappell, 2017; Connelly & Clandinin, 1988; Shulman, 1987) to determine their next teacher move. Connelly and Clandinin (1988) describe this practice as "a particular way of reconstructing the past and the intentions of the future to deal with the exigencies of a present situation" (p. 25). In other words, teachers rely on their past experiences and their understanding of the learning goals to dynamically make decisions (Chappell, 2017; Connelly & Clandinin, 1988).

Conceptual Framework

The ways in which teachers harness their personal practical knowledge to facilitate mathematical discourse within entrepreneurial design-based contexts is largely unknown. Therefore, we approach this work knowing that entrepreneurship presents a complex situation to navigate as the mathematics is embedded in the students' chosen real-world contexts (Belcher et al., 2024) and is, thus, difficult for teachers to anticipate and prepare for ahead of time. Additionally, teachers may encounter student-generated ideas that are beyond their expertise, creating yet another challenge to supporting students' mathematical discourse within their context of interest. To that end, grounded in an understanding of the diversity of student ideas in entrepreneurial design-based projects and the appreciation of a teacher's personal practical knowledge, we seek to better understand teacher discourse during small group check-ins. This paper, therefore, explores the research question: *How do teachers draw upon their personal practical knowledge to facilitate discourse when supporting students to learn mathematics through entrepreneurial design projects?*

Methods

This investigation contributes to a broader study on the design and analysis of the D&P Challenges in STEM framework. In this section, we examine the D&P Challenges in STEM framework and the specific challenge used in this study, provide a brief overview of the context and participants, and

detail the processes of data collection and analysis. An exploratory qualitative methodology was used to understand teachers' use of personal practical knowledge for facilitating mathematical discourse during check-ins with small groups of students. An analytic framework based on TDMs (NC^2ML, 2024) was used to analyze video data through a priori coding followed by a thematic analysis (Creswell & Poth, 2018).

D&P Challenges in STEM Framework

The D&P Challenges in STEM (Confrey et al., 2019; Krupa et al., 2021) aim to boost student interest in mathematics by embedding standards-aligned mathematics learning within design challenges that culminate in entrepreneurial pitch competitions (Belcher et al., 2025). Each design challenge includes a set of criteria that create a need for the specific mathematics content targeted by the challenge. Students collaborate in teams to brainstorm ideas, research existing solutions, prototype new solutions, test and refine their designs, and ultimately pitch their solutions to a panel of external judges. During the design challenge in this study, students designed a navigation app that lets users plan routes based on personalized criteria not typically offered by existing mapping apps. Students used linear functions to build calculators (in Google Sheets or the Desmos online graphing calculator) that can automatically calculate relevant trip information, including route distance, travel time, and estimated time of arrival, for two prototype routes.

Context and Participants

The study participants consisted of five teachers who attended a professional development experience that ran alongside a student camp. Alexandra, Bess, and Kayla were high school teachers (two beginner mathematics teachers and one veteran science teacher, respectively) and two were elementary teachers (both mid-career). All names used in this paper are pseudonyms. The two elementary teachers were not in attendance for the entire time so their data and names were excluded from this analysis. During the professional development, teachers were provided with the opportunity to practice eliciting student ideas about their entrepreneurial design challenges through check-ins with student groups. There were seven students (rising ninth through 12th grade) divided into three groups. To illustrate some of the interactions, a few student names are provided (e.g., Sadie, Thomas, Lynn, and John).

Data Collection and Analysis

This paper analyzes the 14 video recordings in which the teachers were checking in with student groups. The check-in videos, ranging from 10 to 30 minutes, were memoed (Miles et al., 2014) for evidence of personal practical knowledge (Connelly & Clandinin, 1988) in terms of facilitating discourse (NC^2ML, 2024). The videos were then transcribed and cleaned. Next, a priori

coding (Creswell & Poth, 2018) using a TDMs framework developed by NC^{2}ML (2024) was applied to explore the research question.

Analytic Framework

TDMs are a set of nine teacher moves (NC^{2}ML, 2024) for facilitating mathematical discourse. They consist of (a) different types of questioning, specifically *advancing student thinking* and *assessing student understanding*, (b) intentional inclusion of student ideas, specifically *inviting student participation*, *assigning competence*, *attributing student thinking*, and *orienting to other's work*, and (c) creation of space for students to process and think about ideas, specifically *waiting*, *encouraging in-progress thinking*, and *revoicing* (NC^{2}ML, 2024). Table 1 defines each TDM as used in the coding process.

Table 1

Definitions of TDMs Used in the Coding Process

TDM	Definition
Advancing student thinking	Asking questions that move students beyond their current thinking
Assessing student understanding	Following up with an individual student's idea to have them elaborate
Assigning competence	Publicly praising in a way that is specific to the task and intellectually meaningful
Revoicing	Restating or rephrasing a student's mathematical contribution
Inviting participation	Providing students with the opportunity to share their ideas
Orienting to others' work	Asking students to engage with another student's idea
Encouraging in-progress thinking	Encouraging students to share their thinking about a mathematics problem at any stage in their work
Attributing student thinking	Using students' names to connect their ideas to one another's ideas and the goals of the challenge
Waiting	Providing students with time to process the teacher's questions and think about their responses

Coding Process

In the first round of coding, one exemplar video transcript per teacher participant was selected and a coding team of three researchers each simultaneously applied the TDMs framework (Table 1; NC^{2}ML, 2024). During this first round, the coding team met after coding each transcript to discuss the application of codes and resolve any disagreements (Creswell & Poth, 2018). This initial calibration of the teams' application of the codes aimed to establish trustworthiness, using Eisner's (1991) technique of gaining an "agreement among competent others that the description, interpretation, evaluation, and thematics of an educational situation are right" (p. 112). Once the codebook had

been calibrated, the remainder of the transcripts were equally divided amongst the three coding team members (Creswell & Poth, 2018; Miles et al., 2014). The coding team continued to meet during this process to discuss any questions that arose, further establishing the trustworthiness of the findings (Creswell & Poth, 2018).

Thematic Analysis

The thematic analysis of the codes consisted of looking for patterns in the codes (Saldaña, 2014) that helped explain how teachers draw upon their personal practical knowledge of facilitating discourse when supporting students to learn mathematics through entrepreneurship. Patterns amongst the codes were explored and organized around initial themes. The initial themes were defined by the research team and brought back to the data to check for coherence. After the review of themes, and clarifying the definitions of the themes, the following three themes emerged regarding the intention of teacher discourse moves during check-ins: positioning students as experts, co-designing with students, and pushing students towards an outcome.

Results

In this section, we describe how teachers leveraged their personal practical knowledge during student check-ins, using TDMs to elicit and further student thinking. The section is organized by TDM, ordered by how frequently a TDM was observed (see Table 2 for TDM frequencies). One TDM, *waiting*, is not reported on as the data did not allow for proper analysis. While most of the TDMs supported small group discussions, there were times when the use of a TDM created challenges for teachers to navigate, such as student defensiveness or confusion. Three themes emerged that describe the outcome of using TDMs during check-ins with small groups: positioning students as experts, co-designing with students, and pushing students towards an outcome. The themes demonstrate different ways to support mathematical learning during the Routes Reimagined challenge. When students feel like the experts, their "(positional) identities as knowers and doers of mathematics" (Herbel-Eisenmann et al., 2013, p. 182) are nurtured. Co-designing with students further supports these identities because the teachers are basing their contributions on the students' foundational ideas. Finally, when teachers are pushing students towards specific learning outcomes, they are ensuring that students gain access to the opportunity to learn the targeted mathematical goals. These themes are highlighted within the different discourse moves.

Advancing Student Thinking

Teachers primarily used the *advancing student thinking* move to push students towards an outcome and co-design with students. *Advancing student thinking* sometimes triggered defensiveness in students, posing a challenge for teachers to manage.

Table 2
Frequency of Teacher Move in Teacher-Student Interactions

Teacher Discourse Move (TDM)	Frequency (n=449)
Advancing Student Thinking	219
Assessing Student Understanding	195
Assigning Competence	41
Revoicing	41
Inviting Participation	32
Orienting to Other's Work	27
Encouraging In-Progress Thinking	13
Attributing Student Thinking	9

Note: Many quotations were coded with more than one TDM.

Pushing Students Towards Outcomes

There were multiple instances of teachers using the *advancing student thinking* move to push students towards the outcome of prototyping their navigation app. To design their prototypes, students needed to create sample routes and build functions to show how their app would automatically calculate trip details (e.g., distance, estimated time) for those routes. Kayla demonstrated this push in the following quote:

> This would be leg one with this speed, leg two with this speed, and he has a map, right? Yeah, I think for your pitch you need to make sure that you're being explicit to the judges that you get that if it's automated it's including multiple types of legs and multiple variables.

Kayla leveraged the context and the pitch competition to push students to build their prototype and thus engage with the intended mathematics. In this way, the *advancing student thinking* move had the potential to push students towards the outcome of defining variables and building linear functions through the contextual need to automate estimated time of different legs in the route.

Most teacher-student interactions resulting from an *advancing student thinking* TDM helped students progress in the challenge. However, in some cases, teachers attempted to *advance student thinking* in ways that were not well-received. For example, Kayla, reacting to her group's prototype mapping a route from Domino's Pizza to the summer camp location, exclaimed, "Oh God, no no. Let's pick an interesting area." The students responded defensively

and tried to explain their reasoning for choosing that route, yet Kayla continued to push back. The exchange eventually led to Kayla partially backing down by saying, "y'all had a really narrow spot to look at as an example and I think maybe there's just not that much interesting stuff there." Although Kayla's *advancing* with the small group was an attempt to push students towards the outcome of creating a prototype that would interest the judges, by not drawing upon the challenge criteria or student ideas, she instead created tension that had to be overcome to retain rapport.

Co-Designing with Students

Teachers also used *advancing student thinking* to co-design with students. For example, Alexandra used her knowledge of the community to co-design with her group:

> …usually that's like thirty-five [mph]...And then what about this green section right here? From Dorothea Dix Park to whatever street you said? Martin Luther King? Or Western Boulevard? …Western Boulevard is thirty-five. Okay. So where do we, where do we change? Does this one change here? Once you are on New Bern, I think that is forty-five.

During this interaction Alexandra gathered the students around one laptop and walked through their prototype to show how knowledge of the local community could improve their design. They continued co-designing together, representing speed and distances in their prototype. Acting as a co-designer, Alexandra leveraged the students' unique solution to *advance* their mathematical thinking, creating a need for their linear functions to account for varying speeds.

Assessing Student Understanding

Assessing student understanding was primarily used to position students as experts. Since teachers did not have the benefit of anticipating potential student solutions, they relied on the *assessing student understanding* move to make sense of the work the small groups had been doing. This is evident when Alexandra checks in with a small group of students who are prototyping on their computers:

> So, you have it in Desmos. Can I see it? Okay, let me decipher this. So, let's see. Yeah, I don't know if I can figure this out. I think you have to walk us through. Because these numbers don't have context for me, you know what I mean?

By asking the students to explain the meaning of the numbers, Alexandra leverages their positions as experts of the work to draw out mathematical interpretations of the functional relationships and variables at play in the students' solution.

The *assessing student understanding* move did not always position students as experts. All three teachers asked some variation of the same question

about mathematics, and it often led to student confusion. Alexandra asked: “Did you do any math yet?”, Kayla asked, “Did y’all do the math piece that’s involved?”, and Bess asked, “Do you feel prepared to do the math?” Students did not always recognize when they were doing mathematics and had difficulty responding to these questions. Therefore, TDMs that simply *assessed* whether the mathematics was attended to rarely led to productive conversation. This contrasts with results above where the teachers used the *advancing student thinking* move to further the mathematics within the prototypes.

Assigning Competence

Assigning competence was primarily used to position students as experts. For example, in the following quote, Bess stated her validation and then follows it with a reason for why she thought their idea was “cool:”

> That's really cool, so even like a student driver who’s nervous about, ‘Okay, where is this building, right I can look at it the night before, I'm gonna drive and know how much and what I’m gonna drive.

Teachers typically *assigned competence* by validating students’ ideas, sometimes with description and justification of what they liked, which could help them “recognize strengths in themselves and their peers” (NC^{2}ML, 2024, p. 2).

Revoicing

Teachers primarily used the *revoicing* move to position students as experts, specifically to clarify that they understood the students correctly. Alexandra demonstrated this when she asked, “Who are we marketing this to?” and when the students responded that they were marketing to travelers, she revoiced saying, “So all travelers. Or like people? So, like long-distance travelers?” By restating what the students had said and then asking follow-up questions to clarify, Alexandra showed the students that she was interested in fully understanding their idea.

Inviting Participation

The *inviting participation* move was primarily used to push towards the outcome of increased collaboration. Teachers would ask questions to bring students into conversations when they perceived other students as dominating the discussion. For instance, Alexandra *invited* a disengaged student to participate by asking a targeted question that the student would be uniquely positioned to answer. After a teammate commented that they did not know of anything along their prototype route, Alexandra excitedly asked, “Ohh, Thomas what’s around Eagle School?” This TDM resulted in the student contributing his knowledge of the community and introducing a new leg into the prototype route, which created another opportunity to apply the targeted mathematics of building linear functions.

Orienting to Others' Work

Teachers used the *orienting to others' work* move to position students as experts. Kayla did this regularly, saying things like "Okay, when I left, Lynn was working on calculating the distance of a scenic route," "John was doing the normal route," and "Okay, Sadie was color-coding routes and was finding scenic places." These statements *oriented* the students to each other's work, created opportunities for improved collaboration, and gave them the authority to self-monitor and take ownership of their progress.

Encouraging In-Progress Thinking

Encouraging in-progress thinking was used by teachers to co-design with students. This can be seen in the way that Bess uses "we" and "our" in the following quote:

> The quality of our prototype will only improve. This is just an ideation for you to get an idea. This animation was created using Google Maps street view and we want to do even better than they are.

Bess used phrases like "will only improve" and "just an ideation" to *encourage in-progress thinking,* showing students that their animated prototype was a rough draft to refine for the final pitch. This interaction orients students to an attitude of iteration and continual improvement.

Attributing Student Thinking

There were few *attributing student thinking* quotes, most of which revolved around the visuals connected to the prototypes to position students as experts. In one such quote, Kayla *attributed* the visual efforts to John, "I think John has a real good visual." Then in another, when facilitating a group decision around their visuals, she attributed it to Sadie, asking, "Does Sadie need to scrap her picture at this point or do y'all want her to wait and y'all decide if y'all are going to change view?" Through *attributing student thinking*, Kayla connected the group members to each other's ideas and efforts (NC^{2}ML, 2024) and facilitated collaborative decision-making.

Discussion

By observing how teachers engaged with students during small-group check-ins, this study highlights the ways teachers drew on their personal practical knowledge (Connelly & Clandinin, 1988) to naturally employ TDMs (NC^{2}ML, 2024) in support of mathematics learning within the dynamic context of entrepreneurial design projects. Specifically, the teachers in this study leveraged TDMs to position students as experts of their entrepreneurial designs, engage with the students' ideas as co-designers, and push students towards various outcomes (both mathematical and non-mathematical) that were important for success in the Routes Reimagined challenge. Overall, the analysis

illuminated how teachers could use TDMs to more effectively support mathematics learning amid unique student-generated contexts. These findings demonstrate to professional development facilitators areas to focus on in teacher learning experiences.

While all TDMs were observed during the study, the teachers used the questioning moves (*advancing student thinking* and *assessing student understanding*) more frequently than the moves used for intentionally including students (*inviting student participation*, *assigning competence*, *attributing student thinking*, and *orienting to other's work*) or creating space for students to process ideas (*waiting*, *encouraging in-progress thinking*, and *revoicing*; Herbel-Eisenmann et al., 2013; NC^{2}ML, 2024). Teachers' personal practical knowledge (Connelly & Clandinin, 1988) for these small group conversations consisted primarily of questions and comments that sought to understand what the students were doing and then move students towards the successful completion of the entrepreneurial design project. The less observed TDMs can be essential for empowering students to develop mathematical identities (Herbel-Eisenmann et al., 2013). Therefore, professional development should intentionally include opportunities for teachers to experience and learn to implement these less frequently used TDMs, building them into their personal practical knowledge.

To support mathematics learning and empower mathematical identities, teachers need to understand when mathematics is occurring during the course of a project (Grossman et al., 2019). Since the targeted mathematics is integrated into the context students choose to address in their D&P Solution (Belcher et al., 2024), it can be difficult for teachers to recognize when and how these mathematical concepts are present in students' solutions (Cook & Weaver, 2015). This can lead to teachers feeling discomfort if they believe their students are not learning the intended mathematics. Professional development facilitators and curriculum designers should help teachers explicitly identify how the mathematics is embedded in the design of a project at the various points of the process and ways to deepen student understanding of those mathematical concepts through discourse.

Conclusion

This study set out to explore an under-researched intersection of powerful concepts in mathematics education: how teachers use their personal practical knowledge of discourse facilitation to support mathematics learning within the complexities of a D&P Challenge in STEM. It should be noted that these findings come from a small sample of teachers during a week-long professional development. While these findings are not generalizable, the hope is that they motivate future research specifically around how to build on teachers' personal practical knowledge during small group check-ins where students are generating unique, hard-to-anticipate solutions to complex

problems.

When engaging in the D&P Challenges in STEM, effective student discourse is key to ensuring that collaboration leads to meaningful mathematical learning. Observations of teachers during small group check-ins revealed that while they frequently used questioning moves like *advancing student thinking*, they less frequently used moves that intentionally include students or allow for processing ideas, moves that are crucial for empowering students' mathematical identities. To enhance teachers' personal practical knowledge for these complex interactions, professional development should focus on these less frequent moves and help teachers recognize when mathematics is occurring within a project.

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