Journal of Mathematics Education
2025, Vol. 18, No. 1, 66-83

https://doi.org/10.26711/007577152790182

Challenges of Linking Levels of Classroom and Professional Development when Qualifying Facilitators

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In this study, we investigated whether and how awareness of the distinction between classroom level and teacher professional development (TPD) level develops among novice facilitators during a professional development program for facilitators (FPD). The FPD program is a one-year German Center for Mathematics Teacher Education (DZLM) program for prospective facilitators of mathematics TPD. At the beginning of the program, many of them mainly reflected on their own classroom practices. It is therefore crucial that they extend their reflections to teachers' learning at the TPD level. Thus, our focus is to enable them to link the classroom and the TPD levels and to switch between those two levels while reflecting. Based on data from reflections in group discussions at the beginning and end of the qualification, we observed the statements made by 30 novice facilitators. For this purpose, online video recordings of group discussions were made and transcribed, and the statements were coded using a theory-based category system. The data analysis shows that novice facilitators can be supported in their professionalization through structured activities during the FPD program to shift their focus of reflection from one level to the integration of both levels. In addition, an increase in competence was observed in the statements of individual novice facilitators with regard to a sharpened perception of different requirement situations within the two levels.

Keywords: novice facilitators, teacher professional development, facilitator preparation, three-tetrahedron model, expertise model

Facilitators initiate the long-term professional development (PD) of teachers and accompany, support, and advise them in their instructional development. Therefore, the development of facilitators' didactic competencies is of great importance for practice and research (Prediger et al., 2022a; Zaslavsky & Leikin, 2004). In most cases, facilitators are still working as

teachers themselves and have many skills and a great deal of knowledge about teaching. However, when these experienced teachers become facilitators, they in a sense become beginners again—an experience that can be uncomfortable and often confusing. One issue here is the role change from teacher in the classroom to facilitator working with teachers. What this all means and what challenges this new level entails is the subject of this article. It is therefore important to better understand this critical transition from classroom teacher to facilitator in order to better support their development.

This transition was empirically investigated for different groups of mathematics teacher educators, for example, for mathematics coaches by Chval et al. (2010), for university educators by Dinkelman et al. (2006), and for facilitators who lead TPD by Schwarts et al. (2022). All three studies provide the first indication that a successful transition does not leave classroom teaching experiences completely behind. The transition results in facilitators who integrate reflections at classroom and TPD levels.

However, for facilitators to be able to professionally plan and conduct a TPD program, they need far more subject-didactic skills than just being good mathematics teachers themselves. Rather, a specific didactic is required that addresses the core practices (Forzani, 2014; Grossman et al., 2009) of teachers and offers learning opportunities that enable professionalization at the TPD level. We build on current research (Prediger et al., 2022b; Schwartz et al., 2021) by examining the question of novice facilitators' PD processes. In doing so, we observe the extent to which awareness of the diversity of levels develops among facilitators in the context of a one-year FPD program.

Theoretical Background: Mathematics Teachers become Facilitators

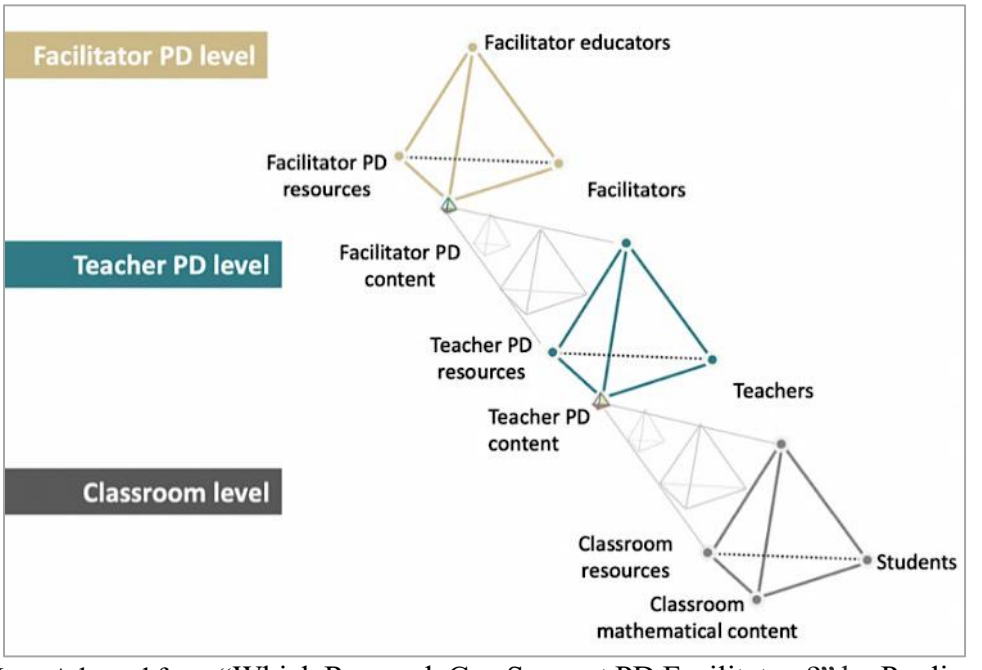
One aim of FPD is to ensure that novice facilitators have the knowledge and tools to successfully plan and implement TPD programs independently. On the one hand, this concerns the orientation towards rather generically formulated design features of effective TPD (Barzel & Selter, 2015; Yoon et al., 2007); on the other hand, the focus is on the content and subject-didactic perspective, as this is where the specific challenges for the design of TPD programs arise.

Professional Skills of Facilitators

Of particular interest is teachers' subject-didactic knowledge: their content knowledge and their pedagogical knowledge. According to Wilhelm et al. (2019, p. 229), a closer analysis of the specific requirements for facilitators reveals that their professional competencies encompass further areas. Compared to teachers, the competencies of facilitators are (1) much more theory-based than practice-based than the competencies of teachers, (2) not exclusively teaching-related but also clearly TPD-related, and (3) networked in relation to individual areas of knowledge.

The three-tetrahedron model (see Figure 1) not only shows the different requirements at classroom and TPD levels–it explicitly distinguishes the FPD level. This level describes the facilitators' professionalization including the implementation and management of TPD programs. In this model, the content, resources, and learners are presented at each level as the components relevant to the classroom, TPD, and FPD levels and their relationship to each other. The TPD level includes all aspects of the tetrahedron at the classroom level as the object of learning. As an extension the FPD level includes the entire tetrahedron of the TPD level as the object of learning of novice facilitators.

Figure 1
Three-Tetrahedron Model



Note. Adapted from "Which Research Can Support PD Facilitators?" by Prediger et al., 2019, *Journal of Mathematics Teacher Education*, *22*, 407–425. CC BY-NC.

Therefore, facilitators need knowledge not only at the classroom level, but also at the TPD level (Prediger et al., 2019; Tzur, 2001; Wilhelm et al., 2019). It is therefore necessary to determine this knowledge, and there are already descriptions and in-depth analyses of the required knowledge and practices at the TPD level (Peters-Dasdemir et al., 2023; Wood & Turner, 2015; Zaslavsky & Leikin, 2004). When determining these knowledge categories, constructs that are specifically required for the promotion of mathematics education are often assumed. These were transferred to the TPD level, such as mathematical knowledge for teaching (Ball et al., 2008). This is also evident in the three-tetrahedron model through the nesting of the tetrahedrons.

Consequently, the practices at the TPD level, for example, are also considered central didactic competence elements (González et al., 2016).

However, it is not enough to view knowledge and practices in isolation, as a key competence for facilitators is the awareness and ability to focus on both levels. This raises the question of connection and demarcation: What references are there, for example, in the materials between classroom and TPD levels? How do these materials differ? Why is it sometimes difficult to distinguish between the use in classroom and TPD levels?

In our further consideration, we will focus centrally on subject-specific didactic TPD knowledge, as we are particularly interested in differentiating and linking classroom and TPD levels. Dinkelman et al. (2006), Chval et al. (2010), and Wilhelm et al. (2019) have already thoroughly elaborated on the specific challenges involved in the transition from a mathematics teacher to a facilitator. For example, Dinkelman et al. (2006) and Chval et al. (2010) identified these challenges as initially strong teacher identities. Wilhelm et al. (2019) described how focusing on their own experiences at the classroom level initially prevented novice facilitators from reflecting on the thinking and learning of other teachers. In these studies, the facilitators gradually learned to think about the TPD level. All these studies characterize the necessary transition between levels not as a complete shift of focus to the TPD level but as a gain by broadening the focus. They generally describe facilitators successfully learning to integrate their reflections across both tetrahedrons. However, the manner of integration has not yet been described in detail, partly because the research focus has been on teacher identities rather than on the uttered content itself.

Schwarts et al. (2022) identified three different modes by which facilitators draw on their teaching experience for the TPD level: (1) implementing a familiar classroom routine at the TPD level, (2) differentiating the classroom level from the TPD level by (2.1) naming essential differences or (2.2) questioning the relevance of a routine for the TPD level, and (3) importing classroom aspects into new TPD routines (e.g., by illustrating arguments with the help of the classroom level). However, these modes require further disentanglement, as they can only be used to a limited extent to investigate the establishment of relationships and connections between the levels.

To date, little is known about how classroom and TPD levels are linked, and which knowledge and which requirement situations are (or must be) associated with each other at both levels. In addition to other theoretically possible links, such as delimitation, integration, or direct reference of the TPD level to the classroom level, so-called "lifting" and "nesting strategies" (Prediger et al., 2019, pp. 412–413) have been highlighted.

To act purposefully in the role of a facilitator, it is necessary to have the competence to perceive and differentiate between different requirement situations at both the classroom and TPD levels (Karsenty et al., 2021). The requirement situations described in the subject-related expertise model provide categories (at classroom and TPD levels) that are necessary to be able to carry out future work as a facilitator professionally (see Table 1).

Table 1
Requirement Situations of the Expertise Model

Requirement situations	Jobs	Pedagogical tools	Categories for thinking and perceiving	Orientations
Description	This refers to typical challenging didactic situations in further PD or in the classroom that facilitators/teachers must master to realize their further PD/lessons.	These tools concern all artifacts and instruments with which facilitators/ teachers design further PD/lessons.	These categories concern the conceptual, non-propositional knowledge that filters the facilitators'/teachers' categorical ways of thinking and perceiving and thus focuses their practices.	This refers to cross-subject or subject-related attitudes of facilitators/teachers that implicitly or explicitly shape the prioritizations and interpretations of the respective requirement situation.
TPD Level ⇕	Stimulating teachers' thinking, selecting content/tasks/media/ methods, diagnosing learning states, moderating joint discussions, supporting and promoting learning processes, and specifying the learning objectives of a PD session.	Tasks, methods, impulses of conversation, guidance or support formats, and research articles.	Cognitive activation, media use, forms of presentation, types and facets of knowledge, networking activities, and bases of understanding.	General attitudes toward the subject matter or learning processes and individual settings constituting good teaching, such as recognizing the relevance of the basis of understanding.
Classroom Level	Stimulating learners' thinking, selecting content/ tasks/media/methods, diagnosing learning levels, announcing learning objectives, moderating plenary discussions, and identifying/ diagnosing/promoting bases of understanding.	Tasks, methods, discussion impulses or supportive formats/booklets.	Cognitive activation, media use, basic concepts, foundations of understanding, connecting representations, and creation of meaning.	Focus on understanding, principle of diagnosis-guided learning, mathematics as interrelated vs. isolated, learner-orientation, interlinking of representations, content-based thinking before calculation, and cognitive activation.

Research Questions

With the help of the required situations, facilitators' competencies can be analyzed over the FPD program. It is assumed that competencies in individual requirement situations become more differentiated and precise during the FPD. The first step is to look at which of the levels they are perceived at all. The next step is to ask whether and how these levels relate to each other or are integrated.

RQ 1: To what extent do novice facilitators differentiate between classroom and TPD levels, and what types of level links can be identified?

RQ 2: What changes in the patterns linking classroom and TPD levels can be observed in the reflections of novice facilitators during the FPD program?

RQ 3: To what extent do novice facilitators use concrete requirement situations when linking levels and can a change in this respect be recognized during the FPD?

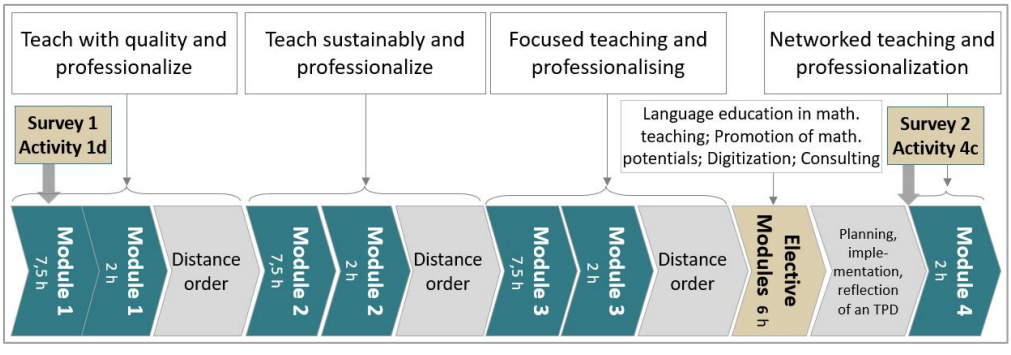
Design of the Study

Framework Data, Structure, and Content of the FPD Program

The FPD program took place in the 2020–2021 school year, with a scope of 60 hours and, due to the coronavirus pandemic, online in the form of four full-day modules with subsequent distance phases. The term "distance phase" describes the time between two face-to-face sessions in which novice facilitators work independently on their own or in small teams and get the order, for example, to try out tasks in their own lessons ("distance order", Figure 2). Two weeks after each of the first three events, an additional online meeting took place in which selected content was revisited in greater depth, teaching experiences were discussed, and open questions were addressed.

After the 3rd module, an elective area was offered in which the 30 novice facilitators (active and experienced teachers at lower secondary level; 21 females and 9 males) attended in-depth mathematics didactics workshops according to their individual interests.

Figure 2
The FPD Program



Novice facilitators discuss in Module 1, for example, the potential of simulations in a methodical three-step process (Thurm et al., 2022) by (1a) working on a concrete problem-solving task themselves (teaching activity), (1b) discussing a teaching video where two students work on this task (from (1a)) with a digital graphic tool (TPD activity to reflect on teaching processes), and (1c) reflecting on a TPD activity initiated by a video of a teachers' discussion during a TPD about the scene in the students' video from (1b) (FPD activity focusing on teacher learning). Afterwards (1d) the novice facilitators were given the following exercise in small groups: "Discuss the potential of such a three-step activity for our main FPD goals. How would you use (and adapt, if necessary) the three-step sequence of activities in a TPD course?" Through intensive preparation in (1a) to (1c), we hoped to encourage novice facilitators' reasoning at classroom and TPD levels.

Between Modules 3 and 4, groups of three novice facilitators each (4a) planned and (4b) conducted their own one-off three-hour TPD course. In the context of Module 4, they were asked (4c) to reflect on this with three other novice facilitators who had observed their TPD, using two open questions: "What did you notice positively in the TPD course you carried out/observed? Which open question would you now like to discuss together?"

Data Gathering

To visualize the development of the novice facilitators' competencies over the FPD program, online discussion rounds in groups of six people each were recorded at the beginning (activity 1d) and at the end (activity 4c) of the FPD. The groups were given a module-related discussion assignment. The following measures were taken to ensure the comparability of the results of the group discussions at the two survey times: (1) The discussion assignments were formulated similarly in terms of structure and procedure. (2) The discussion assignments related to specific current module content and questions of Module 1 (1d) and to the reflection of a self-planned and implemented TPD course (4c). (3) Both discussion assignments offered the opportunity to refer to all the requirement situations of the expertise model at both the TPD and classroom levels. (4) The timeframe available for the discussion is comparable. At the beginning, the discussions took place as part of a group discussion lasting up to 30 minutes; at the end, the novice facilitators spent 15 minutes reflecting on their own planned and completed TPD course and 20 minutes on another TPD course attended by another group. (5) The discussions were recorded as online video recordings. (6) A discussion leader and a recorder were determined in advance to structure the discussion.

To enable reflection (activity 4c) on their own TPD in the form of a discussion, three novice facilitators each observed the planned and implemented TPD of their three colleagues. All online discussions subsequently conducted by the novice facilitators were recorded using an online video recording, submitted, and later prepared for analysis by transcription and anonymization. Cases in which data from novice facilitators were missing due to absences or technical problems, either at the 1st or 2nd survey, were not included in the analysis, which reduced the number of novice facilitators studied to 18. We will show three coding examples from this group of novice facilitators and later focus on a total of five novice facilitators (pseudonyms Anne, Elfi, Elisabeth, Leon, and Nora) in their different development within the FPD program when presenting the results.

Data Analysis

For the coding of the sections, a coding scheme was developed from theoretical principles to check the individual statements of novice facilitators for reference to classroom and/or TPD levels, as well as to requirement situations of the expertise model at TPD and classroom levels. A statement could have none, one, or several references at the same or at different levels.

Empirical Insights into Novice Facilitators' Reflections and Their Growth

Table 2 shows that in the statement in activity (1d) by Elisabeth, the requirement situations *jobs* (moderator), *pedagogical tools* (role cards), and *orientations* (experience with role-playing games) were coded at the TPD level and the requirement situation *orientations* (stubborn pupil/technology-averse tender girl) at the classroom level.

Table 2
The Case of Elisabeth: Coding of Requirement Situations

	Requirement situations at **TPD level** 0 [no reference] 1 [with reference]				Requirement situations at **classroom level** 0 [no reference] 1 [with reference]			
Elisabeth Activity (1d)	Jobs	**Pedagogical tools**	Categories for thinking and perceiving	Orientations	Jobs	Pedagogical tools	Categories for thinking and perceiving	*Orientations*
"Well, I also think, from my experience with role-playing games with adults, **it's very important that you describe the roles well as a moderator.** So that you perhaps even say beforehand using **role cards** - so now play a *stubborn pupil* or now play a very well-adjusted - now to serve the role clichés now - *technology-averse tender girl* or something."	1	**1**	0	1	0	0	0	*1*

To examine the links between classroom and TPD levels in the statements (RQ1), the coding scheme was additionally provided with codes. Table 3 shows that in the statement in activity (4c) by Elfi the code *several levels* was used to record whether a statement had references to no level (0), to one of the levels examined (1), or to two of the levels (2). This code can theoretically be linked to all codes, as a reference to the expertise model can be accompanied by no, one, or several level references. The code *level links* was used to code whether the novice facilitators established a link between classroom and TPD levels with a statement (1) or not (0). Based on the viewing directions in the three-tetrahedron model, a distinction was made between code *top-down* (TPD to the classroom level) and *bottom-up* (classroom to TPD level). Statements in which both perspectives were adopted were coded as *reciprocal*. The requirements for facilitators differ from those for teachers. Statements in which the relationship between the two levels occur in the form of delimitations between the levels were coded as *delimitation* (Table 4).

In Elfi's statement (Table 3), she addresses both levels and links them by delimiting them. Within the codes for level linkage, only one coding with (1) is possible in each case and only if an existing level linkage has also been coded with (1).

Table 3

The Case of Elfi: Coding of the Type of Level Links

	Levels 0 [no] 1 [one] 2 [two]	**Links** 0 [no] 1 [yes]	**Type of level links** 0 [no reference] 1 [with reference]			
Elfi Activity (4c)	Several levels	Level links	Top-down	Reciprocal	Bottom-up	**Delimitation**
"So as a question - what is my role as a facilitator for the groups? And that means teacher groups. I would differentiate again between student and teacher groups. Because we are in **a different role**. They are more independent than pupils. At least the ones you had sitting there were really good."	2	1	0	0	0	**1**

In Table 4, five of Anne's utterances are printed that can exemplify most of the codes. We observe that she has (a) more reflections, (b) more extended reflections to both levels, and (c) more integrated reflections, with even all four kinds of links that we identified in the whole data corpus.

Table 4

The Case of Anne's Growth in Reflection: Coded Example Utterances

Activity 1d in the beginning of the FPD (2 utterances in total; only TPD level is addressed)	
Only TPD level addressed	"But then I find that our PD program here is [only] three hours long. For me, the problem is rather that I already have far too many ideas for these three hours. Then you would have to leave out a lot of other things. So that would be […] a challenge for me."
Activity 4c in the end of the one-year FPD (22 utterances in total: 8 x only on one level, 4 x two levels without linking; among the 10 with links: 2 x delimitation, 5 x bottom-up links, 2 x top-down links, 1 x reciprocal links)	
Delimitation	"I know this situation [time problem in class]: somehow you have to teach everything, as a teacher you have to give the input or you have to do this and that, then you somehow lose the focus and […] my feeling here [in the TPD] is different […], it is better [in a TPD] to really go into depth on one thing and to unravel the different facets that make a topic new, instead of covering so many things."
Top-down link	"[After the TPD], they should take a look at their [school] curriculum material and choose the good materials they have [got in the TPD]. Of course, this is of no use for a one-time TPD program."
Bottom-up link	"Or bring student errors on worksheets to the event [the TPD] - as an assignment. Four weeks before, they should then collect good student errors and bring them [to the TPD]."
Reciprocal link	"If I take the topic 'find algebraic expressions and equations' [for the TPD]- why don't we develop a little teaching sequence for [dealing with the suggested model]. Like a lesson plan or something- how could you work with it? Then offer a second model- then that has more potential. Ok, if you realize that can't be represented in the model. But that's not a problem. These are two insights. First, I can't represent them in the model- that means I learn the limits of the model. Secondly […] we can reflect on how to formulate good prompts, there would have been the potential for a long conversation."

Note. Adapted from "Learning to integrate reflections on the classroom level and teacher professional development level." by Bardy et al., 2023, *Proceedings of the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)*, 4750–4757.

The interrater reliability for the final coding scheme (14 codes) was tested using a random selection of 10 transcribed statements with three raters before being applied to the entire transcribed data material and yielded substantial agreement (Fleiss $\kappa = .70$).

Results

The transcribed statements from 18 novice facilitators resulted in 44 meaning units on activity (1d) and 235 meaning units on activity (4c) (compare Figure 2). The difference in the numbers was because the online design of the module meant that a discussion first had to take place during the 1st survey. Due to the corona pandemic situation, the entire group was separated at this time; part of the group was present together on one screen, while another part was at home with an own screen. Only a few novice facilitators knew each other, all were confronted with the online environment for the first time and were still reluctant to comment at the beginning. In addition, some statements could not be coded at this stage, as they did not relate to the required discussion topic. Some novice facilitators were restrained in the 1st survey and made no contributions. However, they were significantly more active in the 2nd activity. In activity (4c) it was also explicitly requested that each novice facilitator say something about the observed TPD. In the following section Table 5 documents the frequencies of the codes for all statements made by the 18 novice facilitators in the group discussions.

Results in Relation to the Ability to Link Different Levels

To examine this aspect of FPD more closely, the number of statements linking TPD and classroom levels, as well as the direction of the links and the addressed requirement situations, were analyzed for all novice facilitators. Regarding the potential of the design of activity (1d) at the beginning of the FPD program, Table 5 shows a remarkably low number of statements relating only to the classroom level. It seems that the design of the sequential activity (1a/b/c) before (1d) was successful in initiating the statements of the novice facilitators in relation to the TPD level already in Module 1. We assume that the thematization of a simulation in a TPD course contributed to this result.

Overall, in the small group discussions (1d) at the beginning, 9.1% of the statements showed a delimitation of levels, and 18.2% of the statements showed top-down transfer effects from the TPD to the classroom level. In only 8 statements (18.2%) did the novice facilitators articulate reciprocal or bottom-up connections.

Table 5

Facilitators' Utterances Coded for Addressed and Linked Levels

	Addressed levels			Two levels linked	Modes of linking two levels			
Total of codes in utterances	Only classroom level	Only TPD level	Classroom & TPD level		Delimi-tation	Top-down	Reci-procal	Bottom-up
Activity 1d n = 44	2 (4.5%)	18 (40.9%)	21(47.7%)	20 (45.5%)	4 (9.1 %)	8 (18.2 %)	4 (9.1 %)	4 (9.1 %)
Activity 4c n = 235	5 (2.1%)	134 (57.0%)	79 (33.6%)	73 (31.1%)	9 (3.8%)	27 (11.5%)	8 (3.4%)	29 (12.3%)

Note. Adapted from "Learning to integrate reflections on the classroom level and teacher professional development level." by Bardy et al., 2023, *Proceedings of*

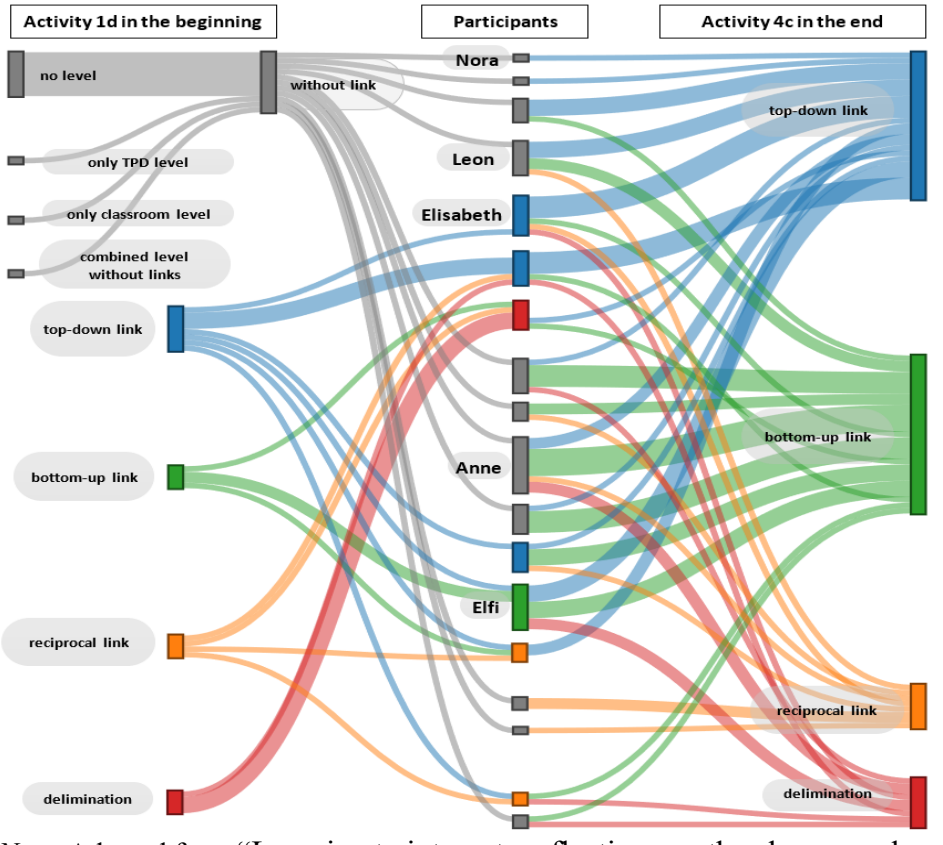
the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), 4750–4757.

Heterogeneity of Growth Pattern

Long-term changes are recorded here by comparing the statements in activity (1d) with activity (4c) at the end of the FPD. The novice facilitators contributed a total of 235 statements with references to levels. The proportion of statements relating only to the classroom level was already low at the beginning and fell even further (from 4.5% to less than 2.1%). In contrast, the proportion of statements referring only to the TPD level rose from 18 to 134 (from 40.9% to 57.0%). The proportion of combined statements in which both levels were addressed fell from 47.7% to 33.6% (even though the number of responses rose from 21 to 79); the proportion of statements linking the levels also fell (from 45.5% to 31.1%). Overall, we interpreted these figures as an initial indication of the intended competence development of the novice facilitators.

Figure 3

Sankey Diagram on the Changes of Links and Modes of Links



Note. Adapted from "Learning to integrate reflections on the classroom level and teacher professional development level." by Bardy et al., 2023, *Proceedings of*

the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), 4750–4757.

In detail, the changes in the focus of reflection in relation to a level link of all 18 novice facilitators are illustrated by the Sankey diagram in Figure 3. In particular, we have mentioned the five pseudonymous names of those who will be discussed in more detail below. The course of the lines makes it clear that the novice facilitators who initially showed no signs of linked reflection in relation to the two levels (e.g., Nora or Leon) predominantly developed top-down and bottom-up links. The novice facilitators who initially articulated only one type of level linkage expanded their repertoire of integrating levels (e.g., Elisabeth, who began with only one top-down linkage, and Anne, who initially articulated only the TPD level); both even expanded their statements to include all four possible level linkages.

Anne and Elfi are taken as examples of two people who differ significantly from each other in their development; their data is shown in Table 6. Anne does not use any level links in her argumentation in activity (1c), whereas Elfi uses several links. At the end of activity (4d), however, Anne uses all level links more often, whereas Elfi does not use the link reciprocal.

Table 6

Development of the Number of Links in the Cases of Anne and Elfi

		Total coded statements	Levels addressed – Anne & Elfi							
			Only classroom level	Only TPD level	Class-room & TPD level	Without linkage	Type of level links			
							Delimitation	Top-down	Reciprocal	Bottom-up
1d	Anne	2	0	1	0	0	0	0	0	0
	Elfi	6	0	2	3	0	0	1	0	2
4c	Anne	22	1	7	14	4	2	5	2	1
	Elfi	25	0	14	10	3	2	3	0	2

Analysis in Relation to the Focus on Different Requirement Situations on Several Levels

As the total number of statements is not yet sufficient to draw a clear conclusion about the reasons for the increase, additional analyses were conducted. For this reason, references on the requirement situations were also compared from the 1st to the 2nd survey. Several requirement situations can occur in one statement, which explains the high overall values.

A closer examination of the available statements reveals that a broader view of the requirement situations goes hand in hand with the FPD program. Activity (1d) shows that only just under half of the novice facilitators commented on *jobs* and *pedagogical tools* at the TPD level and that there was a lack of focus on *categories for thinking and perceiving* (Table 7). In activity

(4c), almost all the novice facilitators mentioned the requirement situations *jobs* and *pedagogical tools* on the TPD level in their statements. These changes described are less pronounced at the classroom level. In the 2nd survey, more novice facilitators were now commenting on *jobs* and *pedagogical tools* at the classroom level (increase also for *categories for thinking and perceiving* and *orientations*).

Table 7

Statements Coded According to Requirement Situations and Levels

Total coded Statements with reference to a requirement situation		**Classroom level**				**TPD level**			
		Jobs	Pedagogical tools	Categories for thinking & perceiving	Orientations	Jobs	Pedagogical tools	Categories for thinking & perceiving	Orientations
1d	$n = 44$	13 (29.5%)	6 (13.6%)	3 (6.8%)	7 (15.9%)	17 (38.6%)	30 (68.2%)	2 (4.5%)	20 (45.5%)
4c	$n = 235$	40 (17.0%)	24 (10.2%)	22 (9.4%)	28 (11.9%)	159 (67.7%)	95 (40.4%)	37(15.7%)	52 (22.1%)
Number of participants with reference to a requirement situation									
1d	$n = 18$	7 (38.9%)	5 (27.8%)	2 (11.1%)	5 (27.8%)	9 (50.0%)	8 (44.4%)	1 (5.6%)	8 (44.4%)
4c	$n = 18$	14 (77.8%)	12 (66.7%)	11 (61.1%)	11 (61.1%)	18 (100%)	18 (100%)	15 (83.3%)	14 (77.6%)

To assess whether the novice facilitators had achieved an increase in their ability to focus on different requirement situations, the number of references to the requirement situations was an important indicator. The data shows that the novice facilitators' references to requirement situations were higher at the TPD level than at the classroom level. A comparison of the sum of references per novice facilitator shows a significant increase ($p < .01$, Hedge's $g = 1.09$) in the mean value.

Overall, the statements made by the novice facilitators in the 2nd survey increased significantly and showed a broader spectrum of requirement situations at the TPD level. The requirement situations *jobs* and *pedagogical tools* at the TPD level are mentioned by all the novice facilitators at the end of the FPD program. In contrast, the requirement situations *categories for thinking and perceiving* and *orientations* are less pronounced, although there is also a clear increase in relation to the 1st survey. The increased didactic and methodological depth of the statements, which is reflected in the increase in the total number of references to requirement situations per novice facilitator, is further evidence of a specific increase in the novice facilitators' subject-didactic skills.

Table 8 shows that the novice facilitators Anne and Elfi developed very differently about the requirement situations expressed. Anne, who did not use any level links in the 1st survey, initially made a few comments. In the 2nd survey, she used all four level links and commented on all requirement situations at both levels. Elfi can expand her different level links during the

FPD, whereby she addresses all requirement situations at both levels in the 1st and 2nd survey.

Table 8
Focused Requirement Situations in the Cases of Anne and Elfi

	Classroom level				TPD level			
References to the requirement situations *n*: # Statements	Jobs	Pedagogical tools	Categories for thinking & perceiving	Orientations	Jobs	Pedagogical tools	Categories for thinking & perceiving	Orientations
1d Anne *n*=2	1	0	0	0	0	0	0	0
Elfi *n*=6	2	1	0	0	2	5	0	2
4c Anne *n*=22	1	3	2	4	14	6	4	4
Elfi *n*=25	2	1	3	1	14	8	3	12

Combination of Level Links and Requirement Situations

Anne and Elfi link the requirement situations differently. In activity (1d), Anne did not show any level linkage. In activity (4c), Anne linked classroom and TPD levels in relation to the requirement situations (Table 6): reciprocal (*jobs* is linked), top-down (*pedagogic tools* is linked), bottom-up (*categories for thinking and perceiving* is linked), and delimitation (*orientations* is delimited). Anne also used different types of linking for different requirement situations.

Elfi expanded the types of links she expressed from activities (1d) to (4c) overall. In activity (1d), she linked only the requirement situation *jobs* and *pedagogical tools* bottom-up. She also showed this link in activity (4c). Her number of references to requirement situations increased (Table 8). In activity (4c), she linked the requirement situation *jobs* and *pedagogical tools,* not only bottom-up but also top-down. In addition, a link can be recognized by delimiting *categories for thinking and perceiving*.

In contrast to Anne, Elfi linked the requirement situation *jobs* top-down and not reciprocally. In addition, *categories for thinking and perceiving* are not linked bottom-up, as in Anne's case, but as boundaries. In activities (1d) and (4c), Elfi linked *jobs* and *pedagogical tools* bottom-up. For her, this seems to be a stable type of linking for these requirement situations. Overall, the statements of these two novice facilitators show that the same requirement situations are linked differently. The stability of the bottom-up linking of *jobs* and *pedagogical tools* is also an initial indication that specific links are used for the same requirement situations.

Conclusion

Working as facilitators requires specific demands that go beyond their subject-didactic skills at the classroom level and relate in particular to the TPD

level (Prediger et al., 2019; Tzur, 2001). The three-tetrahedron model demonstrates the complexity of these extended skills. Facilitators of mathematics TPD all over the world need specific skills to be able to analyze classroom and TPD levels in relation to the situational challenges. They must develop a differentiated conscious view of both levels to be able to establish links between the levels. This should become a central content of FPD programs. The three-tetrahedron model was successfully used to analyze statements made by novice facilitators in relation to the levels. It has been shown that with the help of the FPD program, the differentiated focus on both levels (classroom and TPD levels) could be increased by specifically addressing the specific skills of novice facilitators in differentiating and linking the two levels. These findings are relevant for FPD programs in general.

The evaluations also showed that a focused consideration of classroom and TPD levels does not translate one-to-one into a necessary linking of the levels. Rather, the ability to link both levels in a meaningful way represents an additional PD step for the novice facilitators. The patterns of how novice facilitators establish a link between the levels can be described (RQ1) as delimitation (relationship between the levels through differences), bottom-up (from classroom to TPD level), top-down (from TPD to classroom level), and reciprocal (simultaneous treatment of levels). These four concepts were newly developed and empirically proven in the context of an FPD program. At the beginning of the FPD program, all types of links occur with a similar frequency, whereas at the end of the FPD, a predominance of unidirectional links can be seen (RQ2). This tends to indicate a PD step in which the novice facilitators expand and simultaneously consolidate their familiar linking patterns, although individual differences can be observed.

Overall, the novice facilitators were also able to make a greater number of links at the end, and for the majority, the proportion of statements with level links also increased compared to the proportion of statements without level links. In the discussions, it became clear that they showed a greater sensitivity for the different levels, which is why they were also more flexible in their ability to link requirement situations on the levels (RQ2, RQ3). It can also be seen that the novice facilitators use different types of linking for one and the same requirement situation and that some novice facilitators remain stable in their use of one specific type of linking for one requirement situation over the FPD. These are interesting new findings with regard to the content design of argumentation processes in TPD programs that facilitators should consider.

The basis for the successful FPD of novice facilitators is the experience and skills gained from their professional practice as teachers. During the FPD, the novice facilitators now undergo learning processes at the classroom level, for example, when they deal with the subject-related requirements for high-quality, sustainable, focused, and networked teaching, thus deepening their subject-didactic skills at the classroom level. On the other hand, there is an expansion of subject-didactic knowledge and professional competencies at the TPD level. Even if the requirement situations differ from the classroom level,

the expertise model can be used to create analogous categories in terms of subject didactics and subject matter.

Although TPD and classroom levels are intertwined in the modules, their ability to focus on requirement situations at the TPD level is more strongly influenced than for requirement situations at the classroom level. This is a new finding. The FPD program carried out generally supports the novice facilitators in recognizing and describing requirement situations at TPD and classroom levels to make them usable for TPD courses they have planned themselves. Personal prior knowledge and experience at TPD and classroom levels also play an important role in statements on TPD content.

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Acknowledgement
The research is conducted within the DZLM, the German Center for Mathematics Teacher Education. We thank the Department of Education in Hamburg for funding the FPD program QuaMaMo.