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Exploring Students' Mathematical Creativity for the Concept of Enlarging and Shrinking through a Technologically Conjecturing Teaching

Pi-Jen Lin
Pei-Yu Chang Liao
National Tsing Hua University, Taiwan

This study explored students' creativity performance in a regular mathematics classroom where a teacher incorporated IT tools into creativity-directed conjecturing teaching. The participants were the instructor and 21 sixth graders. The primary data included the cases and the conjectures generated by the individuals, groups, and the entire class, collected from ten lessons. The data analysis for students' creativity in the creativity-directed conjecturing tasks and teaching involved classifying conjectures, establishing a coding framework, calculating the percentage of the whole-class conjectures, and scoring each conjecture with a score of 10, 1, and 0.1 for fluency, flexibility, originality, and elaboration, the four components of creativity. Additionally, the scores of each student's creativities were calculated. The study results indicated that IT tools integrated into creativity-directed conjecturing teaching were an approach for developing students' creativity in regular mathematics lessons and developing students' concepts of enlarging and shrinking by distinguishing distorted from non-distorted geometric shapes. The concept of enlarging and shrinking is beneficial to developing students' elaborative thinking when students are required to create geometric Picasso patterns with the IT tool integrated into the conjecturing teaching.

Keywords: IT tools, creativity, creativity-based conjecturing teaching, enlarging and shrinking, elementary school

Creativity is at the forefront of 21st-century skills. Creativity for all students has become a prominent goal of school education nationwide. However, there has yet to be an agreement on the definition of mathematical creativity (Mann, 2006; Sriraman, 2017). Creativity in schooling differs from that of mathematicians (Leikin, 2009). Absolute creativity in mathematicians is associated with great works at a global level, while relative creativity refers to discoveries by a person in a specific reference group (Leikin, 2014). It is necessary to distinguish absolute creativity from relative creativity. This relative creativity has two implications for school education. One is that creativity can be nurtured through school education instead of inherited. The other is that students' solutions or insights in solving challenging mathematical problems can be indicators of creativity (Liljedahl & Sriraman, 2006). Our design of facilitating students' creativity in mathematics classrooms adopts a relative perspective of creativity, emphasizing the essential relationship between learning and creativity through school education. Thus, we adopt Kaufman and Beghetto's (2009) mini-creativity (mini-c) as the definition of creativity. Mini creativity (Mini-c) is involved in "constructing personal knowledge and understanding" (Kaufman et al., 2016). Mini-c for teachers is most likely to meet in schools. Students' creativity can be developed from mini-c in everyday lessons.

To foster creativity, offering students adequate cognitive challenges is essential (Nadjafikhah et al., 2012). A mathematical challenge is a mathematical difficulty, that a person can and is willing to overcome (Leikin, 2014). It requires teachers to give students multiple-solution tasks to engage them, to guide students in asking suitable questions, to learn how to make conjectures, and to justify their conclusions, as mathematicians do. However, mathematics instruction often lacks the challenging tasks to promote students' creativity. Among the various instructional approaches used in mathematics classrooms, we adopt the IT tools incorporated into creativity-directed conjecturing teaching (CCT) as the framework for designing mathematics lessons. The conjecturing teaching aims to develop students' deep understanding and creative thinking (Lin, 2018). In this study, we focus on how a teacher incorporated IT tools into conjecturing-based teaching to identify and characterize students' creativity in regular mathematics classrooms, where students engaged with enlarging and shrinking at the primary school level.

Conceptual Framework

Mathematical Creativity

Creativity is one of the critical competencies of the 21stcentury that allows us to adapt to fast changes in all fields of life (PISA, 2021). It has become one of the goals of reformed curricula across countries. Teachers and researchers need to identify what creativity means and how it can be facilitated in mathematical classrooms. The definition of creativity varies with different researchers. For instance, Kaufman and Beghetto (2009) identified four types of creativity: (a) Big-Creativity (Big-C) or extraordinary, which is found in people who are acknowledged over time; (b) Pro-creativity (Pro-c) or expert creativity, which is found in professionals who are excellent in their field and are acknowledged as experts; (c) Little-creativity (little-c) or ordinary creativity, which is found in everyday activities and can be recognized by others; and (d) Mini-creativity (Mini-c), which is defined as "the subjective self-discoveries that are part of the learning process" and consists of "new and personally meaningful insights and interpretations that may not be considered creative by other people" (Kaufman, et al., 2016, p. 395). Mini-c emphasizes the importance of the relationship between learning and creativity. Creativity in school education could start by enhancing mini-c in everyday lessons. According to Haylock's suggestion, any definition of mathematical creativity in teaching must refer to mathematics, teaching, learning, and creativity (Haylock, 1997). Thus, we focus on how mini-c might be nurtured in regular mathematics classrooms, where students might perform, and how it could be identified.

The components of mathematical creativity in the measuring framework of Leikin (2009), which are adopted as the common indicators of creativity in Guilford's (1967) and Torrance's (1974) models, consist of fluency, flexibility, and originality. Elaboration is not included in Leikin's framework. In Leikin's studies on mathematical creativity mostly use open-ended problems, problem-posing tasks, or investigation tasks (Leikin, 2018). The multiple solutions are only for a specific mathematical problem. Thus, elaboration cannot be revealed in the solutions of solving a typical mathematics problem. Furthermore, Leikin and Lev (2013) developed an evaluation framework of creativity, including the originality, fluency, and flexibility components with a decimal base 10-1-0.1 scoring scheme. The number of solutions usually is used to measure fluency. These types of solutions are the indicators of measuring flexibility. The evaluation of originality requires both relative and absolute examination of the solutions. The relative examination refers to his/her reference group that produced the answer. A solution's creativity (Cr) = fluency x flexibility x originality. Leikin's (2013) framework for evaluating creativity has three features: (a) It can determine the quality of different solutions a student uses. (b) The decimal-based scoring can reflect the problem-solving process and product. For example, A student's flexibility score of 31.2 points means he proposed 6 = (3 + 1 +2) solutions. Of the solutions, 3 solutions are in three different categories, 1 solution in a similar category but has a minor distinction (1 point), and 2 stands for two conjectures almost identical to one of the three categories (2 x 0.1= 0.2 point).

In the current study, students will engage in conjecturing-based tasks rather than problem-solving tasks. Students will generate conjectures rather than solutions in conjecturing-based tasks. Thus, the scoring schema will code the conjectures instead of solutions. For this reason, we need to adjust Leikin's framework. Two adjustments will be made. One is that elaboration will be included in the framework since mathematical conjectures, which can be true or non-true for all cases, are to be produced. The true conjecture can be generalized to infinite cases, while a non-true conjecture can become a true conjecture once it is restricted to a specific domain. The other is that group-conjecture and whole-class-conjecture spaces will be considered in the framework. The group conjecture space is the basis for flexibility scores, while the whole-class conjecture space is the basis for originality and elaboration scores. The framework for evaluating creativity by engaging in conjecturing tasks is indicated in Table 1.

Table 1

Framework of Evaluating Creativity Performing in Conjecturing Tasks

Fu	Fx	Or	El	Cr
1 per conj.	Fx_i=10, different categories in group-conj.. Fx_i=1, same category but not similar in group-conj.. Fx_i =0.1, similar conj. in group-conj..	Or_i =10, in class-conj., P < 15%. Or_i =1, in class-conj., 15% < P < 40%. Or_i =0.1, in class-conj., 40% < P	El_i =10, a true conj., in class-conj.. El_i =1, a non-true conj. turns into a true conj., in class-conj.. El_i =0.1, a non-true conj., in class conj..	$Fxi \times Or_i \times Eli$
n	$\sum_{i=1}^{n} Fxi$	$\sum_{i=1}^{n} Ori$	$\sum_{i=1}^{n} Eli$	Cr=$\sum_{i=1}^{n} Fxi \times Ori \times Eli$

Fu: fluency; Fx: flexibility; Or: originality; El: elaboration; Cr: creativity
Conj: conjecture.
n: The number of conjectures.
$p = (m_j/n) \times 100\%$, m_j: the number of students generated the jth conjecture.

Creativity-Directed Conjecturing Teaching

CCT, as referred to in the study, is the instructional activities to develop, support, or evaluate creativity skills. CCT is rooted in the perspective that creativity drives knowledge development (Leikin & Elgrably, 2022). The tasks engaged in CCT require three features: (a) Open: Openness can provide multiple opportunities to produce original ideas (Lin, 2018). (b) Mathematical challenges: The difficulty falls within the zone of proximal development (Vygotsky, 1978). Thus, igniting learning occurs. The difficulty creates a cognitive imbalance. (c) High cognitive demands (Stein et al., 2000): Conjecturing tasks demand a high cognitive process, including observing, discovering, testing, checking, and justifying. The conjecturing processes can support and reinforce the development of mathematical creativity.

CCT is based on students' conjecturing cognitive processes with five stages, including construction, formulation, validation, generalization, and justification (Lin, 2018). The construction stage refers to the task given by the teacher, creating cases by individual students and collecting them into group cases by organizing orderly. In this stage, group activities are required. Group cases are collected from the individual cases through collaborative works, and errors that occur during individual cases are checked. Besides, classifying the individual cases into different categories is more likely to make one look for the patterns and discover mathematical relationships.

The second stage is to formulate conjectures by looking for patterns (Lin, 2018). The conjectures must be based on group cases. The truth of a conjecture merely relying on finite cases is possible to be refuted. Therefore, group activities are necessary, including checking, classifying, and compiling the individual conjectures. Checking is for examining if each conjecture is correct and based on cases. Classification is required to sort similar individual conjectures into the same category and then become a group conjecture. Conjecting teaching has the potential to develop students' creativity.

The group conjectures generated in the second stage are only proposed based on cases in a group. Thus, the group conjecture needs to be supported by more cases from other groups. The process is called validation, the third stage. Validating a group conjecture means testing if it is supported or rejected with more cases from other groups. It is followed by classifying the group conjectures into whole-class conjectures. These conjectures collected from the whole class are like a related branch of a tree that stemmed from the same root. The process provides the opportunity to enhance students’ internal connection among mathematics concepts and develops students' mini-creativity. It is one of the features of the conjecturing teaching approach (Lin, 2018).

Generalization is the fourth stage, which extends a conjecture to be true for all cases. If the conjecture is valid only for some cases, it must be limited to the domain or condition to be true. If the conjecture is valid for all cases, then its’ statement needs to add "any" and "all" with precise mathematical language. Finally, verification, the fifth stage, refers to convincing others by justifying the propositions generated in the fourth stage with deductive reasoning.

Research Method

Participants

The participants were the instructor (Yu, the second author of the article) and 21 sixth graders, who have engaged in conjecturing teaching for 1.5 years. Yu has participated for two years in a teacher’s professional learning community to support teachers in implementing CCT. Yu was supported by this professional learning community when she created the task for conjecturing teaching.

Creativity-Directed Conjecturing Tasks

The rationale for designing the conjecturing tasks of enlarging and shrinking with IT tools incorporated into the CCT included: (a) to increase students' concrete experience and make sense of enlarging and shrinking, we provide students with the free design for a picture through computers; (b) observing the change of length, height, and area of the zoomed image, comparing to the original shape; (c) to discover the change in length, height, and areas of without distortion (enlarging and shrinking) and distortion between the two images; (e) the angle without change in the zoomed images is required in the design.

Ten lessons took place in the computer room and a sixth-grade classroom. In the computer room, students accessed Google Classroom and Excel tables. Upon entering the computer room, students must download the "Geometry Picasso Standard Edition" file Yu uploaded to Google Classroom. Then, Yu asked them to identify what distorted and undistorted shapes aligned with a class photo presented in the PPT as the Warm-Up activity. A Task Direction was as follows. (1) Create a Picasso design in the frame using 12 geometric shapes (one circle, one square, three rectangles, three triangles, two parallelograms, and two trapezoids). (2) Only five shapes are used in the design. (3) Do not allow overlap and exceed the frame in a design. (4) A creative design should consist of distorted and undistorted designs. Finally, Yu printed every student's work and took a photo with the designer.

After finishing the design, students were asked to fill in the width and height of each shape in the Excel worksheet via computer and then complete a collective worksheet from individual cases. Before leaving the computer room, each group uploaded an Excel worksheet to Google Classroom. The worksheet for Group 7 is shown in Figure 1 for exploring the shrinking and enlarging geometric shapes.

Figure 1

Cases Created by Group 7 for Exploring the Shrinking and Enlarging Geometric Shapes

Shapes	Enlarge /shrink	Side of new	Side of origin	Ratio of side (new: origin)	Height of new	Height of origin	Ratio of height (new: origin)	Area of new	Area of origin	Ratio of area (new: origin)	Ratio of side x ratio of height	Angle of new	Angle of origin
Rectangle B	Shrink	2.34	3.99	0.59	1.76	3	0.59	4.12	11.97	0.34	0.34	90	90
Triangle E	Shrink	2.97	5.99	0.5	1.98	3.99	0.5	2.94	11.95	0.25	0.25	55	55
Circle M	Shrink	3.02	3.99	0.76	3.02	3.99	0.76	28.64	49.99	0.57	0.57	x	x
Parallelogram L	Enlarge	5.61	3.99	1.41	7.04	5	1.41	19.75	9.98	1.98	1.98	69	69
Triangle F	Enlarge	8.61	5.99	1.44	4.29	3	1.43	18.47	8.99	2.06	2.06	33	33
Rectangle C	Distortion	6.48	5.99	1.08	0.64	2.01	0.32	4.15	12.04	0.34	0.34	90	90
Triangle F	Distortion	3.48	5.99	0.58	2.59	3	0.86	4.51	8.99	0.5	0.5	45	33
Parallelogram K	Distortion	8.71	5.99	1.45	4.09	3	1.36	17.81	8.99	1.98	1.98	44	65

Data Collection and Data Analysis

The primary data included individual, group, and whole-class cases and conjectures collected from personal, group, and whole-class work during the ten lessons. The data collection also included the tasks and the verbatim transcripts of classroom observations.

The data analysis included five steps. (1) Classifying individual, group, and whole-class conjectures. In Figure 2, four students in G7 made a total of 7 individual conjectures (the 4th column) and then collected them into four group conjectures (the 3rd column) and four whole-class conjectures (the 1st column). (2) Creating a coding system, classifying the whole-class conjectures into six categories. (3) Calculating the percentage of each category of the whole-class conjectures. (4) According to the framework for measuring conjectures in Table 1, each conjecture was scored 10, 1, and 0.1 for flexibility, originality, and elaboration, respectively. For instance, the four students put forward 2, 2, 1, and 2 conjectures, respectively. Since the conjectures of S2-5, S9-2, and S18-4 were proposed by more than 15% of students (the 1st column), their originality gets 1 point. The other conjectures were generated by less than 15% of students; thus, originality was 10 points. (5) Calculating the scores of each student's fluency, flexibility, originality, elaboration, and creativity, respectively, as shown in the (Cr) column of

To ensure the reliability of the fluency, flexibility, originality, and elaboration scoring, following the framework for measuring creativity in Table 1, each of the two raters scored the conjecture individually and then compared their scores. The agreement of inter-raters reached 98%. The disagreement was resolved through discussion until a consensus was reached.

Figure 2
Scoring G7's Conjectures as an Example

G7's Conjectures				Fu	Fx	Or	El	Cr	Each Student Cr
Class	%	Group	Individual						
C4: The ratio of reducing is less than 1, and the ratio of enlarging is greater than 1	17	G7-2 True fractions appear only when zoomed out	S2-5 True fractions appear only when zoomed out	1	10	1	10	100	1100
C2: When zoom in /out, side of image: side of origin = height of image: height of origin.	7	G7-3 When zoom in and out, side of image: side of origin = height of image: height of origin.	S2-6 Side of the image: side of origin = height of image: height of origin, and the ratio is equal.	1	10	10	10	1000	
C1: The angles will be the same when the origin is proportional.	17	G7-1 After the origin is enlarged or reduced, the angle of the image is the same	S9-2 Angle of image: angle of origin=1:1	1	10	1	10	100	1100
C2: When zoom in /out, side of image: side of origin = height of image: height of origin.	7	G7-3 When zoom in and out, side of image: side of origin = height of image: height of origin.	S9-7 Side of the image: side of origin = height of image: height of origin.	1	10	10	10	1000	
C1: The angles will be the same when the origin is proportional.	17	G7-1 After the origin is enlarged or reduced, the angle of the image is the same	S18-4 After the origin is enlarged or reduced, the angle of the image is the same.	1	10	1	10	100	100
C2: When zoom in /out, side of image: side of origin = height of image: height of origin.	7	G7-3 When zoom in and out, side of image: side of origin = height of image: height of origin.	S19-3 Side of the image: side of origin = height of image: height of origin	1	10	10	10	1000	2000
C6: When the origin is proportional, the ratio of the area of the image to the origin equals the ratio of length x the ratio of height.	1	G7-4 Side of image: origin: height of image: origin, multiplying the two ratios will be the ratio of the area of the image: origin.	S19-8 Side of image: origin: height of image: origin, multiplying the two ratios will be the ratio of the area of the image: origin.	1	10	10	10	1000	
		G7-Total Scores		7	70	43	70	4210	

Results

Creativity-Directed Conjecturing Teaching for Shrinking and Enlarging

Students were seated in groups of 4. In the construction stage, students started sharing their Picasso creations completed in the computer classroom. They checked and looked for the patterns of distorted or non-distorted geometric shapes. Before collecting individual cases into group cases on a worksheet, each student in each group must mutually check the correctness of the area and ratio of geometric shapes. It is followed by sorting and sequencing the order of cases. The number of cases, classification, and order differed among groups. For example, Figure 1 shows that the G7 generated eight pairs of shapes. They were divided into two categories: distorted and non-distorted.

In the formulation stage, each student worked individually to look for patterns based on the group cases. Then, each student took turns sharing their conjectures. Before sharing, Yu offered students four remarks: (1) Raising and reading your conjectures aloud. The conjectures worksheet must be seen by each member in your group. (2) Specifying where the conjecture was based on the group worksheet. (3) Asking: "*Do you agree?*" "*Are there any questions?*" If your conjecture needs to be modified or added, please mark it with a red pen. (4) Asking: "*Is there any conjecture that is the same as mine?*" At this time, the individual conjectures were classified into different categories. Yu asked students to organize their conjectures to help them understand other's discoveries and avoid repeatedly reporting in a group. This also prevented students from being distracted and saved time for group reports.

During group discussions, Yu patrolled among groups to listen to students' discoveries, supported students, encouraged them to put forward more numbers and categories of conjectures, reminded them of the way of reporting when the speaker's voice was too low, etc.…. Yu played different roles in facilitating students' group discussions.

In the validation stage, Yu led students to share, classify, and sequence the group conjectures. Each group began to report the number of group conjectures. Nevertheless, Yu first asked G4 to share the conjecture: "*The ratio of the length of the enlarging/shrinking image to the origin is equal to (the ratio of the height*." Then, other groups were asked to post similar conjectures on the blackboard. G7 posted a conjecture in which "*In zooming in and out, the length of the image :the length of the origin = the height of the image: the height of the origin, the ratio is equal* ", as shown in the 2nd column of Figure 2.

Classifying group conjectures in the third stage is isomorphic to classifying individual conjectures in the second stage. Students took advantage of classifying skills in the second stage by observing and learning Yu's classification practice. Six whole-class conjectures were formulated after each group shared and categorized the group conjectures. The whole-class conjectures were only supported by the finite cases of the class; thus, Yu helped students to generalize the whole-class conjectures to be endorsed for more cases. In addition, Yu assisted students in revising and elaborating their language usage. She also helped students to understand the meaning of a conjecture becoming a proposition via a constraint and a restricted domain, which turned into the premise of the proposition. The following scenario is how Yu assisted students in refining the mathematical language of conjectures (lines 109-110), understanding the premise of conjectures (lines 111-113), and how to use constraints to make a conjecture to be true for all cases.

107	T:	How do we discuss the relationship between these two undistorted shapes? What conditions does it have to be added in?
108	S22:	Equal ratios.
109	T:	Okay, does a geometric shape have many sides? However, what you are doing is definite, so what should you say about the two sides?
110	S8:	The corresponding sides.
111	T:	Okay, shall we write clearly? What should we change to? In the fourth grade, you all have learned two congruent figures we call corresponding sides, correct?
112	S19:	So, we need to end the corresponding length and corresponding height in the conjectures.
113	T:	Excellent; here, the zoom-in and out is a premise to ensure the truth of a conjecture, and the corresponding side is the elaborated language, which is necessary for refining this conjecture.

In the justification stage, to let students see the relationship among the six whole-class conjectures, Yu began by asking students to classify them. The students divided the six whole-class conjectures into four categories: (1) The angle of the zoomed image is equal to that of the origin, (2) The zoom-in and-out shape is proportional to the length of the original shape, (3) The ratio of the enlarged shape to the origin is greater than 1, and the ratio of the shrank shape to the origin is less than 1, and (4) The area of the enlarging or shrinking shape is a multiple of the area of the origin. It is followed that Yu asked students to sequence the order of these conjectures for justification. Students replied: Both categories (1) and (2) were related to the definition of enlarging and shrinking shapes, so do not need to justify. Moreover, category (4) can justify category (3). S19 justified, "*The area of the enlarging or shrinking shape is a multiple of the area of the origin*." described as "*In enlarging or shrinking, the base length of the imaged triangle must be a multiple of that of the origin*; *the imaged triangle's height must be a multiple of that of the origin*. *Thus, the area of the imaged triangle is a multiple of the origin. Likewise, other shapes as examples have similar justification*."

Students' Creativities Performing in Enlarging and Shrinking Tasks
Students with Excellent Creativity

The result of the study indicates that students were excellent in creativity; they must perform well in the four elements. However, the highest score in one of the four elements does not guarantee the highest score in creativity. The scores of individual creativities are shown in Table 2. The top

5 students in creativity were S19, S16, S2, S9, and S13. S19 was the best in creativity. She had fewer conjectures, but she put forward 10 points of flexibility, originality, and elaboration for each conjecture.

Table 2
Students with Excellent Creativity

Group No.-Student No.	Fluency	Flexibility	Originality	Elaboration	Creativity
G7-19	2	20	20	20	2000
G5-16	3	20.1	21	30	1110
G7-2	2	20	11	20	1100
G7-9	2	20	11	20	1100
G6-13	3	20.1	30	12	1101
G1-15	8	42.2	6.2	60	420

S15 ranked first in fluency, flexibility, and elaboration, but creativity was not the best due to his low score of originality. He was the one who put forward eight individual conjectures in the whole class. Of the conjectures, except for the two conjectures of distorted images, the other six conjectures focused on the relationship between the image of enlarged/shrank and origin. He used analogical reasoning when proposing a conjecture of the enlarged image. It resulted in creating a similar conjecture of the shrunk image. For instance, when proposing "*the ratio of the enlarged image is greater than 1*", he also suggested "*the ratio of the shrank image is less than 1*". As a result, his fluency score was 8 points, ranking the best in the whole class. Each of the two pairs of conjectures was similar, with 1 point, and his flexibility score was 42 points. The flexibility of each conjecture related to the distorted image scores scored 0.1 points. S15's flexibility score was 42.2. Likewise, Table 2 shows that S13 was the best in originality.

Group Creativities

The data in Table 3 reveals five findings:

1. The students' fluency index in the zoom-in and out tasks is to put forward at least three conjectures. Table 3 shows that 54 conjectures were put forward by the 21 students in the whole class, 11 of which were not cases-based or incorrect, and 43 were correct cases-based conjectures, an average of 2.54 conjectures. In the class, the fluency in the G1 was the highest, with 15 conjectures.
2. G6, with 70.2 points, was the best in flexibility. G6 put forward 7 categories of conjectures. For example, both "*the ratio of the shrunk image is less than 1*" and "*the ratio of the enlarged image is greater than 1*" belonged to the relationship between the ratio comparing to 1, so they are in the same category but not similar. Both "*ratio x original side length = side length of enlarged image*" and "*ratio x original height = height of enlarged image*" were similar conjectures.
3. G6 with 54 points was the best in originality. They generated 5 unique conjectures, including: "*The area of the enlarged image must be a multiple of*

the origin." "*The area of the origin must be a multiple of the shrank image*." "*The ratio x length of the original = length of the enlarged image*." and " *The ratio* x *height of origin = height of enlarged/shrunk image*."

4. G1, with 80 points, was best in elaboration; that is, the number of valid conjectures put forward by the students in G1 is the highest, which can be generalized to all situations of the zoom-in and out tasks. The data in Table 3 shows that a high fluency score does not guarantee a high score of originality or elaboration. The fluency score of G1 is higher than that of G6, while the originality score is much lower than that of G6 (8.2 vs. 54). The fluency score of 9 points in G6 is higher than the 7 points in G7, but the elaboration score of 45 is higher than that of 70.
5. The creativity score is not determined by fluency score, but fluency score is necessary for high creativity score. Table 3 shows that students in G7 had the highest creativity score of 4210. It is mainly contributed by three students, all of whom put forward at least one conjecture, which received a total score of 10 in flexibility, originality, and elaboration. The score of fluency of the students in G1 was 10 points higher than that of the G7 with 7 points, but the score of creativity in G1 with 620 points is lower than that of G7 with 4210 points.

Table 3
Group' Creativity in Zoom-in and Zoom-out Tasks Ranked by High to Low

Group	# of Ss.	Amount of group conj.	Amount of individual conj.	Scores				
				Fu.	Fx.	Or.	El.	Cr.
G7	4	4	7	7	70	43	70	4210
G5	1	3	3	3	20.1	21	30	2100
G6	4	4	10	9	70.2	54	45	1421
G1	3	7	15	10	62.2	8.2	80	620
G2	3	3	7	6	51	6	60	510
G4	3	4	6	4	31	4	40	310
G3	3	3	6	4	21.1	4	21.1	120.01
Total	21	28	54	43	325.6 (10p-32 1p-5 0.1p-6)	169 (10p-14 1p-29)	326.3 (10p-34 1p-6 0.1p-3)	9281.21
Ave.	--	4.67	2.54	2.05	15.50	8.05	15.53	44.20

Fu: fluency; Fx: flexibility; Or: originality; El: elaboration; Cr: creativity
Conj: conjecture.
Ave.: Average
10p-32: 32 counts of 10 points.

Students Performing in the Four Elements of Creativity

Table 4 shows four findings:

1. The fluency in Table 4 shows that the majority of (33%) of the students put forward two conjectures, while one student could put forward 8 conjectures. 14% of the students generated invalid conjectures. Invalid

conjectures included incorrect, superficial, or no case-based conjectures. For instance, "*the area ratios of the rectangle and the distorted shape are both 0.2*" is a superficial conjecture.

2. The flexibility in Table 4 shows that 56% of the students generated two categories of valid conjectures. The two categories are: (a) "*the angle of the zoomed image will not change*," and (b) "*the lengths of each side of the zoomed image and the origin correspond to an equal ratio*."

3. The originality in Table 4 shows that only 34% of the students put forward novel conjectures, including: "*The area of the enlarged image must be a multiple of the origin*." "*The area of origin must be a multiple of the shrunk image*." "*ratio* x *length of original = length of enlarged image*," and "*ratio* x *original height = height of enlarged/shrunk image*."

4. The elaboration in Table 4 shows that 38% of the students could propose 1-2 conjectures that can be extended to generalization. For example, G1-S5 proposed six conjectures that can be generalized.

Table 4
Percentage of Conjectures for Each Element of Creativity in Enlarging and Shrinking

Counts and Rates	Elements	0	1	2	3	4	5<
Number of conjectures	Fluency	3(14%)	5(24%)	7(33%)	4(19%)	1(5%)	1(5%)
Category of conjectures	Flexibility	3(14%)	6(14%)	11(56%)	1(6%)	0	0
Amount of 10 points	Originality	14(66%)	4(19%)	2(10%)	1(5%)	0	0
	Elaboration	3(14%)	8(38%)	7(33%)	2 (10%)	0	1(5%)

Discussion

The first result of the study is that mini-c in the context of learning enlarging and shrinking could be developed in regular mathematics lessons. IT tools incorporated into conjecturing-based teaching supported the development of students' mathematics creativity. The tasks included creating a Geometry Picasso design using drawing tools in a computer room, then titling the Picasso Design, and uploading the design and an Excel worksheet to Google Classroom. The tasks offered students the opportunity to create various distorted and non-distorted shapes. The IT tasks not only saved time for measuring the size of the shapes and avoiding inaccurate measurements but also enhanced students' drawing skills via technological tools.

The second result is that students construct the meaningful concepts of “enlarging” and “shrinking” in geometric shapes by distinguishing them from the everyday language of “changing” through exploring the distorted and non-distorted geometric shapes. The non-distorted shapes included an enlarged or shrank image of a geometric shape. In contrast, the design of distorted diagrams in Picasso's Design was intended so that students could likely perceive the proportional relationship of the corresponding sides between two non-distorted geometric shapes. The changes in the lengths of a distorted shape from an origin were not proportional.

The third result suggests that incorporating IT tools into conjecting-based teaching enabled students to learn rich mathematical concepts that go beyond the lesson objectives scheduled in the school textbooks. Students generated six concepts of distorted geometric shapes including: (a) *When the geometric shapes are enlarged or shrunk, the corresponding angles will not change*. (b) *When it is an enlarged image, the corresponding sides' ratio equals the corresponding heights' ratio*. (c) *The length of the enlarged image can be calculated by multiplying the length of the origin by the enlarged ratio.* (d) *The ratio of the shrunk image to the origin is less than 1; the ratio of the enlarged image to the origin is greater than 1*. (e) *The ratio of the shrunk image is smaller than that of the enlarged image*. Finally, (f) *The ratio of the area of the enlarged image to that of the origin is the ratio of the lengths of the enlarged image to the origin multiplied by the ratio of the height of the enlarged image to the origin*. Only the first two concepts are the lesson objectives listed in the school textbooks.

The fourth result suggests that concepts of enlarging and shrinking had the benefit of fostering students' fluent and flexible thinking. Once discovering the corresponding relationship between the proportional side length of the enlarged image, students simultaneously proposed the complementary relationship between the proportional side length of the shrunk image via analogical reasoning. Groups 1, 6, and 7 were the cases of the study. The result of the study indicated that CCT helped develop students' creativity. This study evidenced that the mini-c of Kaufman et al. (2016) can be nurtured in school education. This result could be explained by the CCT designed in the study, which allowed students to observe, distinguish, classify, compare, and sequence different data for discovering multiple conjectures in the first stage. The data created in the first stage of CCT allowed students to initiate fluent and flexible thinking. The data of non-distortion included enlarged and shrank shapes like rectangles, triangles, and parallelograms. The tasks engaging in the second stage of CCT helped students to put forward more categories of conjectures from different perspectives, thus developing students' flexible thinking. The greater the number of conjectures, the better fluent thinking. Moreover, the greater the categories of conjectures, the better flexible thinking.

Finally, the result indicates that the concept of enlarging and shrinking is beneficial to the development of students' elaborative thinking because the teacher provided different geometric shapes when requiring students to create geometric Picasso patterns with this IT tool, including squares, various sizes of rectangles, triangles, parallelograms, and circles. In the fourth stage of CCT, a conjecture student put forward was related to the proportion of the corresponding side of the enlarged image to the origin. This conjecture to be elaborated means that it should be true and extended to all cases. This generalization helped students learn that the circle must be excluded because there is no side in a circle. Likewise, when a conjecture involves corresponding heights, generalizing this conjecture to all cases requires restricting the scope to triangles or parallelograms. This process made students realize the importance and meaning of mathematical prerequisites for a proposition. The process implies that the rubrics of flexibility, originality, and elaboration were based on group-conjecture spaces and class-conjecture spaces. The two collective spaces of measuring creativity in the CCT tasks were different from that of creativity in the problem-solving-based tasks (Leikin, 2014). The elaboration included in the indicators of creativity in mathematics is the contribution of the study, compared to Leikin's previous studies (Leikin, 2018; Leikin & Elgrably, 2022).

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Corresponding Author:

Pi-Jen Lin
linpj@mx.nthu.edu.tw
National Tsing Hua University, Taiwan.
Graduate Institute of Mathematics and Science Education
+886- 3-5622961
Address: No. 521, Nan-Dah Rd., Hsinchu City 300, Taiwan.

Additional Information

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