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Equitable Mathematics Instruction and Learning Geometry through Extended Reality

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This article explores the use of Extended Reality (XR) technologies, such as Augmented Reality (AR) and motion capture, in mathematics instruction, specifically focusing on geometry learning. Our research highlights the challenges students face in learning geometry due to its abstract nature and the potential of XR technologies to create interactive and immersive learning experiences. We emphasize the role of embodied learning and gestures in mathematics education and how XR technologies can support equitable teaching practices. The embodied motions in XR allow for knowledge to be instantiated in physical ways, positioning all students as competent doers of mathematics. Two case illustrations are provided to demonstrate the power of XR technologies in teaching geometry. This article concludes with recommendations for teachers and researchers to incorporate gestures in mathematics instruction and leverage XR technologies to enhance students' understanding of mathematical concepts.

Keywords: augmented reality, motion capture, extended reality, geometry learning, equitable mathematics instruction, mathematical reasoning, embodied learning, gestures

Geometry can be a difficult subject for students to learn as it is an inherently three-dimensional (3D) subject often taught to students in a two-dimensional (2D) manner (Ng et al., 2018). Geometric principles can be perceived by students as abstract and can hinder students' ability to ground the learning in their everyday and perceptual experiences (Goldstone & Son, 2005). How teachers instruct and students learn geometry is changing with emerging technologies. These technologies can allow for new opportunities for abstract concepts to become grounded in students' bodily experience, and for geometry to be taught in three dimensions. This is the gap in the research we seek to address.

The use of Extended Reality (XR), which includes Augmented Reality (AR) and motion capture technologies, has created a need for research and exploration in mathematics education. Technologies like XR are becoming increasingly more common in K-12 schools, as applications are being developed that can reach students at an accessible price. AR overlays through cameras on phones, iPads, and tablets are also increasingly accessible. These can come in the form of cosmetic overlays on social media apps or holograms of shapes in a room. However, since these novel technologies are still new and developing, there are several gaps in the literature. There is a need for more research utilizing these technologies within the educational and classroom contexts. In the field of mathematics education, AR and motion capture have the potential to create collaborative, engaging, and embodied opportunities for students to understand complex mathematical concepts in an interactive and immersive manner (Abrahamson et al., 2020; Lindgren & Johnson-Glenberg, 2013; Washington et al., in press). K-12 students' understanding of geometry can be brought to life with 3D environments (Ahmad & Junaini, 2020; Ng et al., 2018; Palanci & Turan, 2021). The ability to manipulate and work with dynamic systems through gestures that are more interactive and give real-time feedback is a profound affordance of XR (Johnson-Glenberg, 2018; Washington & Walkington, 2024), one that we explore.

We provide commentary to illustrate the power of XR technologies for teaching and learning geometry, focusing on examples from high school students. Although we focus on high school students and geometry for our illustrations, XR technologies that promote gestures can be found across mathematical topics and grade levels (e.g., Ahmad & Junaini, 2020; Demitriadou et al., 2020; Gecu-Parmaksiz & Delialioglu, 2019; Jabar et al., 2022; Pahmi et al., 2023; Palanci & Turan, 2021; Washington et al., 2024, June).

In the following sections we explain how theories of embodied cognition and gestures connect to mathematics education. We then connect geometry teaching and learning standards to specific technologies. Next, we provide an overview of the technologies utilized in this work with sample visual displays of the technological environments. Then, we provide two illustrative examples of high school students using each of the XR technologies. In the first example, we show students building motions to be programmed into the motion capture game. In the second example, we show students using gestures with dynamic mathematical objects. Following these examples, we highlight how gestures that arise when using XR technologies as well as actions on virtual objects can reveal important understandings students have, both to the teacher and to peer collaborators. We further show how students and teachers can build on each other's reasoning and understanding through gestures and actions on virtual objects. We close by giving several recommendations relating to the use of gestures in the high school mathematics classroom. We suggest that the act of gesturing or engaging in action itself may improve students' geometric understanding, which XR environments allow and can provide opportunity for.

Embodied Learning and Equitable Mathematics Instruction

Theories of embodied cognition and learning have become of significant importance for mathematics learning and says that all concepts, including abstract mathematical representations, are understood in body-based ways (e.g., Danish & Enyedy, 2020; Papert, 1980; Wilson, 2002). Branches of embodied cognition suggest that learning is moving and using the body in new ways, like individuals or groups using their arms or their entire bodies to illustrate mathematics learning and aid with the sense making process (Abrahamson & Sánchez-García, 2016; Danish et al., 2020). Embodied learning approaches can leverage the power of *gestures* for mathematical learning (Alibali et al., 2014; Shapiro, 2014). Gesture is defined as personally generated movements of the body (usually the hands) that people use during reasoning or communication (McNeill, 1992). Embodied learning and gestures can help to ground the learning by connecting the body to the mathematical content (Nathan, 2022).

Using embodied learning approaches can support equitable mathematics teaching by elevating students' movements and gestures and giving students new ways of participating in mathematics. Students can make connections between action- and body-based representations of mathematical ideas and more traditional ones (e.g., written work, formulas). Motion, such as the manipulation of virtual dynamic figures and gesturing, allows for the students to reflect their knowledge and experience mathematics in a physical way that can be seen and shared with others and that positions all students as competent doers of mathematics (Danish & Enyedy, 2020). Barsalou (2016) suggests that one defining feature of conceptual processing from an embodied perspective is the use of flexible, context-dependent representations. The National Council of Teachers of Mathematics (NCTM) Research Committee (2015) has set forth aiming to "change perceptions about what it means to do mathematics" (p.139) and the way mathematical learning is perceived. They describe a shift from seeing math as following procedures to obtain the answer to being about problem-solving, reasoning, and curiosity. This broadening of the conceptualization of mathematical ability and recognition of mathematical skills in diverse manners can highlight the knowledge that students have that is visible in their actions and gestures as they engage with XR environments.

Geometry and Technology

The *Principles and Standards for School Mathematics* cover proving geometric theorems and making geometric constructions (NCTM, 2000). An essential concept in the reasoning and proof strand is "Using technology to construct and explore figures with constraints provides an opportunity to explore the independence and dependence of assumptions and conjectures" (NCTM, 2019, p. 78). The primary NCTM (2015) teaching practices addressed here are tasks that promote reasoning and problem solving, the use and connection of mathematical discourse, the facilitation of meaningful mathematical discourse, and support of productive struggle in learning mathematics.

Embodied approaches to instruction have begun to recognize that mathematics can be powerfully taught by integrating the use of learning technologies. Dynamic Geometry Systems (DGS) are software-based tools that allow learners to construct and manipulate mathematical objects like shapes and figures, and research suggests that their use can significantly improve math achievement (Chan & Leung, 2014). Motion capture technologies (e.g., Smith et al., 2014) can allow learners to use their bodies to demonstrate mathematical relationships by instructing learners to make particular movements and then tracking their skeletons to see if they did. Mathematical action technologies are used here to promote cycles of mathematical exploration: creating, testing, revising, and proving mathematical conjectures; along with presenting and connecting multiple representations (Cullen et al., 2020). We present two cases of extended reality for geometry learning. These cases are meant to be illustrative and are not generalizable or applicable to all classrooms. The first is from a motion capture game (Walkington et al., 2022) that asks students to justify geometry conjectures and the second utilizes GeoGebra 3D (Hohenwarter & Fuchs, 2005) within Augmented Reality goggles.

Overview of Technologies

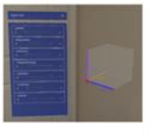
The primary technologies we discuss in our article are motion capture and AR. *The Hidden Village* (THV; Walkington et al., 2022; Figure 1 is an open-source game for high school mathematics using motion capture (either from a regular laptop camera or an Xbox Kinect™ camera) that allows physical motions to be put into a library to correspond with mathematical ideas and conjectures. THV tracks players' skeletons such that it can direct them to perform physical motions and advance them in the game if they perform those motions correctly. Both students and teachers can write new mathematical conjectures with animations for movements that correspond to that conjecture (e.g., crossing your arms to show vertical angles). Their content can be explored by others through a cloud-based library, pooling knowledge and increasing access to mathematics. Currently, this simulation game has been used to explore different geometric conjectures about space and shape properties (e.g., one high school teacher used it to teach a unit on quadrilaterals).

Figure 1

The Hidden Village Game (left) and the Conjecture Creator (right)

Figure 2

Example Conjectures of 2D and 3D Geometric Figures in GeoGebra for AR goggles

2D Simulation	Example Conjecture	Simulation Photo	3D Simulation	Example Conjecture	Simulation Photo
Parallel Lines	If two parallel lines are cut by a third line, the pairs of corresponding angles are congruent.		Spheres	A plane can only intersect a sphere at zero, one, or infinite points.	
Triangles	The sum of the lengths of any two sides of a triangle must be greater than the length of the remaining side.		Cylinders	Given a cylinder with radius r and height h, the cylinder can be unrolled to include a rectangle with length h and width 2πr.	
Parallelograms	Consecutive angles in a parallelogram add up to 180 degrees.		Prisms	If the length, width, and height of a cube are each doubled, then the volume increases by a factor of 8.	
Circles	The perpendicular bisector of any chord always goes through the center of the circle.		Pyramids or Cones	The volume of a cone is one-third the volume of a cylinder with the same base and height.	

GeoGebra 3D for Augmented Reality is an AR holographic mathematics simulation for the Microsoft HoloLens 2 goggles within GeoGebra 3D, a free Dynamic Geometry System (Hohenwarter & Fuchs, 2005). This simulation projects holograms of figures onto the real world (see Figure 2), which can be interacted with and manipulated by multiple learners simultaneously. Students wear goggles that allow them to see and manipulate the holograms with their hands via hand-tracking technology. These goggles also allow students to collaborate with others when they work with the holograms - they can see and change the same holograms as others in their virtual or physical classroom.

Illustration 1: Motions to be Programmed into Motion Capture Game

In our first case, one male and one female high school geometry student have been asked to create arm motions to go with the geometry conjecture, "Consecutive angles of a parallelogram are supplementary." Figure 3 shows students using *collaborative gesturing* (where one learner's gestures reflect or build off of another's gestures; Walkington et al., 2018) to negotiate the meaning of supplementary angles. The extended reality game they are playing has asked them to come up with a sequence of arm motions that show why the conjecture is always true. These motions will subsequently be programmed into the motion capture game for other students to physically perform and then use to understand the conjecture, using a dragging skeleton interface.

In Figure 3, we see the students copying each other's gestures (Lines 2 and 5). For example, one student demonstrates a pose to show the parallelogram and the other copies the pose. Additionally, students use gestures to propose new ideas. For example, in Line 4, one student proposes using two diagonally parallel lines to represent the parallelogram instead of one diagonal and one horizontal line. The students eventually settle on a series of gestures that show the top two angles of the parallelogram being collapsed into a horizontal line (to show they add up to 180 degrees), after discussing and discarding some other options (Lines 3-5). The parallelogram is first represented by two parallel slanted arms, and then collapses into one horizontal line made by placing fingertips together and arms flat. Much of their mathematical discussion takes place via gesture (sometimes with sound effects like "boom," Line 4) instead of via speech, showing how gestures can be a more accessible way for students to communicate spatial mathematical information to each other, compared to formal mathematical descriptions and vocabulary.

This illustration shows how students can communicate via gesture and learn from each other's gesture for the purpose of coming to an agreement or a mathematical conclusion. The task of creating arm poses to be input into the game's Conjecture Creator facilitates new kinds of embodied interactions with technologies. Students proposed final sets of created motions for the game can also be used as a tool for the teacher to assess their mathematical understanding. Further, our research that this illustration came from shows that creating such motions for an extended reality game can improve and deepen students' understanding of mathematics (Walkington et al., 2018).

In Figure 4, we see the students from Figure 3 discussing and entering their collaboratively created poses into the pose creator in THV. These poses are then visible and playable for other students in this cloud-based game. In Figure 4 (P1), you can see the students recounting and repeating their previously created poses (Figure 3) in front of the computer pose creator system (P6). Below the students discuss how to input their poses created for their parallelogram conjecture, discussing how to slant the computer arm positions to reflect their own arms (Lines 1-10, P2- P3). In Figure 4, you can see a group next to their computer game (P4) that allows other groups of students to see and answer the student created and game entered gestures (P6), questions (P5 and P7), and answer choices (P7) in their own gamified learning experience.

Figure 3
Two High School Students Coming Up with Poses to Go with the Parallelogram Conjecture

1. S6-29: We can do something like.
   (S6-29 gestures both arms diagonally parallel, S6-34 gestures both arms to make the bottom of a parallelogram)
   (S6-29 gestures both arms diagonally parallel, shifts one arm vertically, then both diagonally parallel to show sides, S6-34 gestures both arms to make vertically parallel lines)
2. S6-34: This is the parallelogram, it's slanted, and if you go to like this to this it equals one eighty it's a line. So it's
   (S6-29 echos S6-34's previous gesture, S6-34 gestures with elbows out and fingertips together to show the bases of a parallelogram, S6-29 echos this gesture)
3. S6-29: Or we could just do this to this and then that. This is one eighty.
   (gestures arms horizontally parallel, gestures one arm horizontal with the other arms hand touching the first's elbow to show an angle/vertex, S6-34 repeats previous bottom base pose)
4. S6-34: No no cause the two angles have to make one so we could do slanted. And then. Boom.
   (gestures vertical parallel lines, then diagonal parallel lines)
5. S6-34: Cause the parallelogram cause the consecutive angles are supplementary, it equals one eighty the line
   (gestures bases and diagonal parallel lines (2x), S6-29 echos the second diagonal parallel lines)
   (gestures with elbows out and fingertips together to represent the top base, gestures both arms outward to represent the bottom base)

Figure 4

Two High School Students Inputting Their Poses to Go with the Parallelogram Conjecture

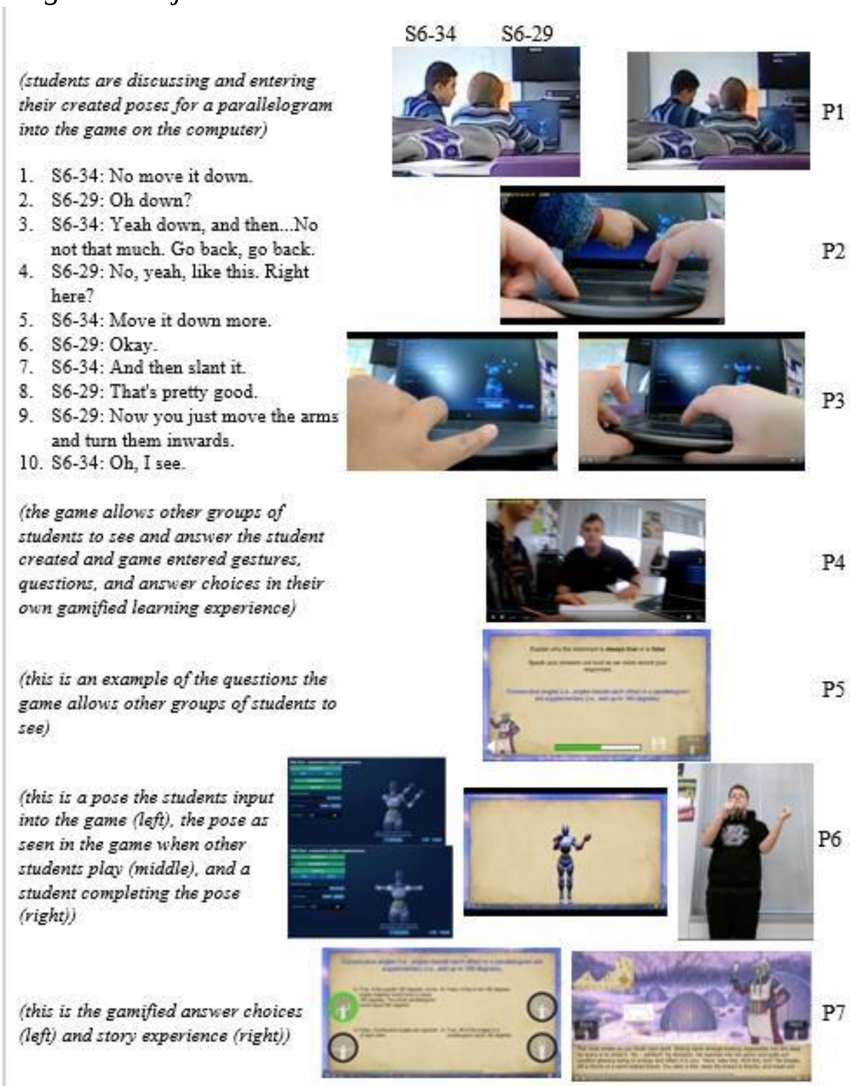


Illustration 2: Using Gestures with Dynamic Mathematical Objects

GeoGebra 3D, here built for the Microsoft HoloLens 2 AR goggles, projects holograms of geometric figures onto the real world. It is a dynamic geometry system where learners see translucent 3D holograms in front of them and can grab and interact with these holograms directly with their hands. Although our illustration has students utilizing HoloLens goggles, students can use GeoGebra 3D with Oculus Quest goggles, smartphones or iPads, or computers. Being able to directly manipulate dynamic, technology-based objects using hand gestures may offer unique affordances (Washington et al., in press) for students gaining a deep understanding of shapes and space.

In Figure 5, two female high school students are using GeoGebra 3D through AR goggles to explore the conjecture: “The volume of a cone is one-third the volume of a cylinder with the same base and height.” The cone appears as a translucent purple figure in front of them, with the cylinder as a translucent white figure that appears around it. Before the transcripts began, the two high school students had interacted with the cylinder and cone, resizing them and observing the results and relationship between them. The cone and cylinder move together, getting taller, shorter, fatter, or thinner, but holding the same proportional relationship.

What is striking in this illustration is that rather than relying on formulas and definitions, the two students instead use their intuition to confront the conjecture, calling upon embodied ideas like cutting a cone out of a cylinder (Lines 5-8, P5-8), and melting the cone to see how much of the cylinder it fills (Line 9, P9). These ideas are further embodied through a series of gestures (Lines 9-11, 13-15, P9-P12), which are distributed across the learners and used to show dynamic transformations of the cone and cylinder. We see clear strengths in the students’ understanding of mathematical ideas when paying attention to their gestures around and within the hologram. We see their understanding of the positioning of the base and the height of the shapes, and the meaning of the 1/3 proportion. We also see their deep understanding of volume as being how much “stuff” a 3D figure is made up of, regardless of how it is organized or arranged.

Figure 5

Two High School Students Discussing “Removing of or Melting of a Cone” Inside the Cylinder

5. S210: you just kind of take a good portion of it out.
   (gestures by rotating hands around the shape)
6. S210: And then but the bottom still has cle like the cylinder did. So, they have the same base, and they have the same height.
   (gestures both hands flat at bottom, then gestures with fingers from top to bottom, back to top)
7. S210: And I'm thinking of how somebody would have a cylinder, and then they would take a cone out,
   (gestures by "grabbing" the inside of the cylinder to remove some)
8. S210: and they would just have this this cylinder, we would like, the cone be hollow. Part of it, so I'm thinking that it is a cone. Yeah.
   (gestures to the cylinder and cone)
9. S209: Yeah, because, I mean, if you look at it, I was thinking about it as like, if the, if the cone were to melt inside of the cylinder, then it would land about like, I think it landed about right here, which looks like 1/3 of the of the cylinder.
   (gestures by showing an approximate 1/3 line on the cylinder)

P5

P6

P7

P8

P9

In Figure 6, as the episode continues, the students use perspective-taking strategies and adjust their positioning to stand next to each other so that they are viewing the holographic figure from the same angle (Line 11, P11).

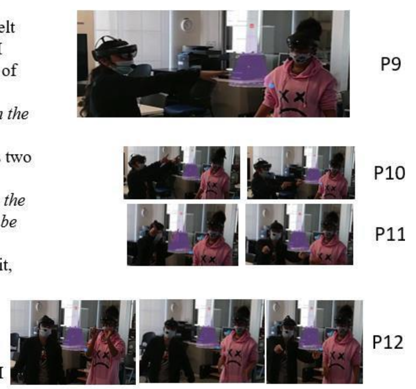
Specifically, S209 moves over and stands beside S210 to see things from her perspective. This makes it possible for the students to point at the same spot on the shape and be visible to their partner in the same position (Washington et al., 2023). These motions help to collaboratively create meaning and understanding. Further, the students copy each other's hand gestures (Lines 10-15, P11-P12). One student demonstrates a motion to show two triangles that fill the open space around the cone within the cylinder, and the other copies the gesture. Much of their mathematical discussion takes place through gestures that are in tandem with or in place of speech. This shows how gestures can be a more accessible way for students to communicate spatial mathematical information to each other, compared to formal mathematical descriptions and vocabulary.

Figure 6

Two High School Students Discussing "The Cone's Area and the Space Around It Within the Cylinder Looking Like Two Triangles."

9. S209: Yeah, because, I mean, if you look at it, I was thinking about it as like, if the, if the cone were to melt inside of the cylinder, then it would land about like, I think it landed about right here, which looks like 1/3 of the of the cylinder.
   (gestures by showing an approximate 1/3 line on the cylinder)
10. S209: So, I would say yeah, and like I mean, there is two other triangles, right here.
    (gestures by pointing fingers from top of cone to the base of the cone/cylinder, adjusts positioning to be next to S210, repeats motions)
11. S209: I don't know how you see it, like how you see it, like, like they go down.
12. S210: Like that yeah.
13. S209: There's upside down triangles.
14. S210: I see what you're doing.
15. S209: You can fit two more in there. So yeah, yeah, I think it is true.

(S210 echos S209 gesture, S209 repeats triangular motion)



Catalyzing Change in Mathematics via Extended Reality

Elevating students' movements and gestures through XR-enhanced learning environments provides new ways of participating in mathematics that highlights students' intuitions about shape and space (Danish & Enyedy, 2020; Hunnicutt et al., 2024; Papert, 1980), are visible to their teachers, and promote collaborative, embodied reasoning. The exploratory and collaborative nature of these XR tools, activities, and the focus on gesturing, can build student knowledge and agency by enhancing problem-solving skills and conceptual understanding (Danish et al., 2020; Nathan et al., 2014; Walkington et al., 2023). In the illustrations, students engage in meaningful discourse as they discuss concepts using speech, actions, whole body movements, and gestures. We note that these cases are not necessarily generalizable to all classroom situations and are intended to illustrate the potential of these technologies. This is an important limitation. In addition, another limitation is that the technological setup needed for these innovations may not be possible in more typical learning environments.

As highlighted in *Catalyzing Change* (NCTM, 2019), reasoning, proof, and modeling “represent broader approaches that cut across the content areas and provide opportunities for teachers to enhance understanding” in ways that connect with equitable teaching. These activities expanded upon what counts as mathematics and the ideas present in NCTM’s (2018) *Catalyzing Change in High School Mathematics* equitable teaching practices. These practices can make meaningful changes in equity and access for mathematics teaching including types of activities that promote reasoning and problem posing, representational mathematical concepts with connections, collaboration, discussion, verbalization of thinking, manipulatable materials, and opportunities to embody the learning (Alibali et al., 2014; Nathan et al., 2014; Nathan, 2022; Shapiro, 2014; Walkington et al., 2019).

Gestures can be observed and built upon in the high school classroom with or without technology. However, as stated in NCTM’s (2019) *Catalyzing Change in High School Mathematics*, we believe that “these newer developments [e.g., dynamic geometry systems] should be part of the geometry learned by all high school students” (Page #?). Activities that embody and extend mathematical understanding, enhanced through technology, can help high school students and teachers by creating opportunities for equity and access through embodied learning. They can expand who is seen in a classroom as being good at mathematics or able to participate in mathematics discourse.

Recommendations

We close by giving several recommendations based on our illustrative examples relating to the use of gestures in the high school mathematics classroom. We highlight broader implications and suggest potential areas of research, along with stating our own directions for future research.

1. Teachers and researchers can observe students’ gestures as a way of assessing their mathematical understanding and recognizing their strengths (Alibali et al., 2014). Students can be encouraged to gesture more by teachers gesturing or using technologies that provide opportunities for gestures.
2. Students can create gestures to correspond to different mathematical ideas in technology-enhanced systems (Walkington et al., 2024). Having more than one gesture for the same idea may often be appropriate. For example, some gestures for “angle” might highlight the two rays, while others might highlight the space in between them.
3. Students should be encouraged to propose ideas to each other and to the class through gestures. Teachers should explicitly acknowledge and discuss

students' gestures as part of the information they are communicating. These gestures can be connected with mathematical vocabulary.

4. Teachers and researchers should use mathematics activities that allow students to use gestures to control digital mathematical objects directly (Walkington et al., 2023; Washington et al., in press). This could be as simple as dragging a vertex on an iPad using an index finger. Associating these "control" movements with mathematical operations may deepen students' understanding.

Future research could examine how such embodied learning activities can be effectively scaled into classrooms, and how teachers can be a central part of the embodied design process. Future research could also examine how gesture as a modality could be integrated into informal and formal classroom assessments. Finally, future research could examine automated detection of gestures through image recognition and its integration into classroom learning. Future directions our team is engaged in include conducting comparative studies where we examine the affordances of AR compared to other modalities through which geometry can be taught, and using more scalable VR technologies to support students' understanding of geometry. Further, we plan to explore how physical movement influences mathematical learning and reveal reasoning patterns by developing an Augmented Intelligence System (AIS) to analyze data from student gameplay and physical motion. The AIS alerts researchers to key cognitive events in real-time, prompting in-depth understanding of students. We plan to look at diverse geometry students playing an educational video game to identify "cognitive shifts" that indicate significant learning insights or challenges. The findings will inform models of embodied mathematical reasoning and support the design of learning systems that enhance STEM education through embodied cognition, particularly benefiting underrepresented and diverse student groups.

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