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Engaging Students in Making Artificial Intelligence Tools for Mathematics Education: An Iterative Design Approach

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This paper presents conceptualization and operationalization of a three-stage iterative design cycle aimed at involving students in making Artificial Intelligence (AI) tools for mathematics education. Grounded in the making of rule-based systems, technology development theories and child as protagonist methodology, the design emphasizes students' involvement in AI making across three stages: task design, participatory actions, and consequences. An ongoing study with twenty high school students highlights the potential of these iterative stages in making a rule-base system. The system functions based on its simulations of students' reasoning processes regarding: (a) hourly changes in the position of the vertices of a 2D shape to form a new shape, and (b) displayed transformation processes, including geometry modeling and constraint satisfaction. Findings from this project, revealing the operationalization of the first iteration of the conceptualized design, are presented. This covers description of students reasoning process in the form of steps, constraints, AI and mathematics concepts. In subsequent iterations, students will refine these descriptions into logics, inferences, and encoding to be simulated by the system. It is recommended that educators operationalize the initial design iteration to inform further AI-making iterations in mathematics education.

Keywords: artificial intelligence, mathematics education, iterative design cycle, student involvement, geometry modeling

The integration of making, an iterative process of designing, building, and refining tools, systems, or representations that bring ideas into tangible or functional forms, has attracted growing attention in STEM education (Blikstein, 2018; Halverson & Sheridan, 2014; Hillman, 2011; Kafai, et al., 2014; Tucker-Raymond & Gravel, 2019). Almost half of teacher education programs in the United States have integrated making courses (Cohen et al., 2017). Notably, the mathematics education programs at Montclair State University (Akuom & Greenstein, 2021) and the University of Maine (Dimmel et al., 2021) include courses that emphasize iterative design cycles for making tools. Often, these courses engage prospective mathematics teachers in the making of tools that utilize either continuous physical properties (analog) or discrete binary logic (digital) to perform specific mathematical tasks. In one of their lessons, Dimmel and colleagues stated they engaged five preservice mathematics teachers who enrolled in their course to develop the Sunrule. They described the Sunrule as an analogue technology that its shadow plane relies on the sunlight to casts shadow which are models of mathematical multiplication.

Given the shift in technology-making from conventional analogue and digital applications to Artificial Intelligence (AI) formats, there are growing societal concerns, and workforce demands that necessitate both educators and students to develop skills in AI-making (Nti-Asante, 2024). These concerns are emphasized by the National Council of Teachers of Mathematics (2024) and the U.S. Department of Education (2023), which have highlighted the need for teachers, particularly mathematics educators, to take an active role in the making of AI tools. Contrary to the making of analogue and digital tools, AI-making is the process of designing, encoding, and iterating systems that simulate human reasoning, learning, or decision-making through rule-based logic, machine learning, or neural network architectures. Although a few studies in STEM Education have adapted technical AI-making conceptual frameworks to engage students (Ng et al., 2023), there remains a notable gap in similar adaptations to involve students and teachers in AI toolmaking within mathematics education.

This paper addresses this gap in three ways. It presents a review of the existing conceptual frameworks from technical AI fields for making AI tools. Secondly, it conceptualizes how they can be pedagogically tailored to engage students in the making of AI tools for mathematics education. Lastly, a revelatory case study is presented to demonstrate the operationalization of the initial stage of this conceptual framework. The paper attempted to address the following two research questions: (1) How can existing conceptual frameworks from technical AI fields be pedagogically adapted to engage students in the making of AI tools for mathematics education? (2) How does the implementation of the initial stage of this conceptual framework, as demonstrated through a revelatory case study, provide insights into its effectiveness and applicability in a classroom setting?

Conceptual Frameworks for Making AI Tools

AI technologies encompass several distinct types of technological systems. These include (a) Rule-Based Expert Systems, which statically replicate human decision-making by employing predefined rules and logic within a specific domain; (b) Machine Learning, which is trained on datasets to recognize patterns and make decisions, with the capability to improve over time by adapting to new data; (c) Deep Learning, a specialized branch of machine learning that utilizes multi-layered neural networks to model complex data, excelling in tasks such as image and speech recognition; and (d) Generative AI, which refers to AI systems that create new content—such as text, images, or music—by learning and reproducing patterns from existing data.

The making of machine learning (ML) systems comprise engagement with established architecture to develop software often in the form of mathematical models. Mumuni and Mumuni (2023) describe the ML architecture as a structured design that encompasses two essential stages. Mumuni and Mumuni described the first stage as involving: (a) data collection, (b) data preparation and preprocessing, (c) data augmentation, (d) the extraction of useful features from the data, (e) the construction of new features, and (f) the pruning of the generated feature set, known as feature selection. They emphasized the second stage as the training of the data by selecting the suitable machine learning algorithm and the optimization of its related hyperparameters to suit the data leading to model making. Mumuni and Mumuni noted that when these steps are implemented independently, the process is referred to as standalone, whereas a unified approach integrates these tasks into a single, cohesive process. In STEM Education, Sakulkueakulsuk and colleagues (2018) engaged students in Thailand through the standalone approach to make an ML model. Firstly, they engaged the students to generate datasets by identifying physical features, such as texture used to classify mangoes into sweetness grades. Sakulkueakulsuk's team then guided the students to use RapidMiner, a UI-based and graphical interface to drag and drop blocks to create flowcharts that represent the logistic regression machine learning algorithmic process of training their datasets into a model. Although the students did not fully deploy their ML model into a functional AI tool, Sakulkueakulsuk and colleagues stated that students applied their models to determine the sweetness grade of mangoes in three different markets, where the fruit could be auctioned. In another study (Ng et al., 2023) involved 35 secondary school students in creating machine learning (ML) models for AI-driven recycling bins. Following the standalone ML approach, Ng and colleagues had students print on cardboards the images of various types of garbage, such as empty aluminum cans and plastic containers. Students then trained these images to become logistic regression models which signals the class, can or plastic of a trash. Ng and colleagues stated that students pasted these models on a trash bin with these two compartments to signal users whether waste should be sorted into the can or plastic compartment.

Grosan and Abraham (2011) provide a detailed account of the process involved in making rule-based AI systems, also known as production or expert systems. They describe these systems as the simplest form of artificial intelligence, designed to mimic the reasoning of human experts in solving knowledge-intensive problems. For instance, these systems replicate the reasoning processes expert mathematics educator employ to perform addition of numbers as its encoded architectural knowledge base, which serves as the foundation for the system's operations. Grosan and Abraham stated that the process of developing these knowledge bases typically begins with system developers conducting in-depth interviews to uncover this specialized knowledge of experts. For example, they may investigate how expert mathematics educator perform addition. Grosan and Abraham added that this unpacked knowledge is then encoded computationally through programming techniques which transforms them into executable forms such as IF-THEN rules. These are stored as the knowledge base of the system comprising the rule-base (general statement) and database (specific statement). As a case, if one of the unpacked ways a mathematics educator performs addition is: "*If I have a dollar and a friend gives me one more dollar, I will have two dollars*". The system developer may develop this as the knowledge base of the system comprising rule and data bases. As shown below, in the rule base, the system developer program the unpacked reasoning process into general rules comprising logics, Condition (IF) and Actions (THEN) while the database comprise the known fact.

Rule Base: containing the programmed general forms of the known facts

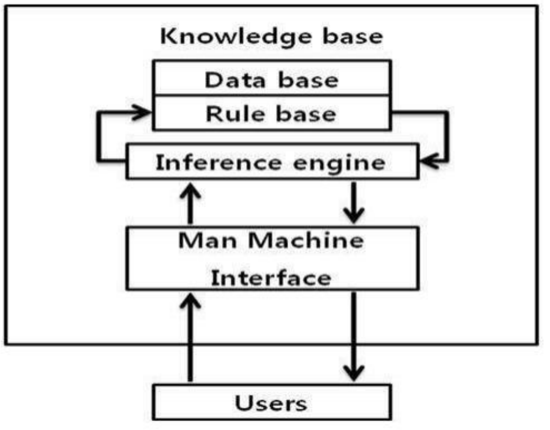
IF current amount = X AND additional amount = Y
THEN total amount = X + Y

Data Base: containing the specific known facts or data

current amount = 1 (dollar)
additional amount = 1 (dollar)

Upon completion, Grosan and Abraham stated that the system developer stores the rule and database on memory cards to be used as the knowledge base of a rule-based system, in this case an expert system for performing addition. As shown in Figure 1, when users interact with this expert system, they use the user interface to input queries such as *"I have a dollar, and a friend has given me one more dollar. How much money do I have?".* These queries are sent to the system's internal inference engine. The inference engine engages in four processes. Firstly, it retrieves keywords from the query and identify IF statements such as "current amount and additional amount" they align with in the rule base. Secondly, it continues by engaging in evaluation by matching aligned query and rule base statements: "IF current amount = X AND additional amount = Y", with their IF statement in the database "IF X = 1 and Y = 1" to ascertain the trueness of the Condition part". In the third stage, it then executes these IFs by matching them with their THEN (Action). Thus, the total amount = X + Y is performed, resulting in total amount = 2. In the fourth stage, it displays this *"You have 2 dollars"* together with the explanation of the processes in the main machine interface. It updates the database by storing the processes.

Figure 1
Components of an Expert System



Note. From "Restoration Aid Expert System for Power Systems" by L. Heung-Jae and O. Jung-Hyun, 2020, *Web of Conferences,152, p.2 (https://doi.org/10.1051/e3sconf/202015203008).* This is an open access article distributed under the terms of the Creative Commons Attribution License 4.0 *(http://creativecommons.org/licenses/by/4.0/).*

In the context of AI education in K-12, Greenwald and colleagues (2021) described the initial step in making rule-based systems as "AI learning." They engaged eight high school students, who served as expert game players, to use pen and paper to represent step-by-step game scenarios that would inform their design of AI gaming environment. Suchman and Trigg (2010) from the fields of science and technology studies described the initial step in making rule-based systems as a craftwork which begins with domain experts creatively writing of the replication of the human behavior into formats for the system to emulate. In science education, a group of researchers designed a rule-based physical science module for students to build knowledge representation system containing facts and rules about mass and gravity, which can reason about the direction of the gravity force given masses of two objects (Akram et al., 2022).

Morales-Navarro and Kafai (2024) acknowledged that while these processes of engaging students in making AI tools offer opportunities for AI learning and traditional computer science practices such as coding and programming, they also present certain challenges. They listed these challenges to include limited opportunities for students to curate the datasets used in ML algorithm making, deploy the ML algorithms into functional tools, and articulate their learning outcomes after participating in the AI-making process. As illustrated in Table 1 below, I present a conceptual framework that aligns the approach for developing rule-based expert systems with educational research methodologies and theoretical perspectives on technology-making.

This alignment addresses the identified challenges and highlights their potential application in mathematics education.

Conceptualization of the Stages for Making Rule-Base Systems

Theoretically, the stages for making rule-based systems in Mathematics Education that are presented in this paper are grounded within socio-cultural learning theories for developmental analysis and its consequences conceptualized for technology development analysis (Hillman, 2011; Wertsch,1991). Hillman (2011) stated that the developmental analysis of technological tools involves engaging the child in all phases of technology's development. He noted that these phases include examining the transition from initial processes to the final product, its use, and subsequent revisions. Hillman emphasized that this approach helps uncover the nature and essence of technology as it evolves and transforms. Concerning the consequence, Hillman stated that as learners participate in this analysis, they engage in various joint activities and internalize the effects of working together while acquiring new strategies and knowledge about technology.

To overcome the challenges with engaging students in the making of AI as outlined by Morales-Navarro and Kafai (2024), I grounded the study within the Child as Protagonists (CP) methodology (Iversen, et al., 2017). Iversen and colleagues stated that the goal of CP is to give children voice in technological design by: (1) describing the objective of the child's participation as the main agent in driving the design process and thereby developing skills to design, (2) describing their participatory action in carrying out the complete design from process to product, and (3) reflecting and measuring the outcome of their participation.

To conceptualize the alignment of this theoretical framing and methods with the stages involved in making rule-based systems for mathematics education, the following process needs to be addressed. Firstly, the initial two components of the methods are operationalized to facilitate the developmental analysis of AI by both teachers and students. This engagement provides a structured opportunity for in-depth exploration and understanding of the rule-based system's development. Secondly, the description of the consequences achieved by students and teachers through these methods is congruent with the theoretical framing of consequences. This alignment shows how the stages for making rule-based systems integrate educational and theoretical outcomes. Consequently, as shown in Table 1, each stage in the making of a rule-based system in Mathematics Education encompasses a cyclical process involving task design, participatory actions, and consequences. This iterative cycle ensures that each phase builds upon the preceding one: task designs inform participatory actions, and the outcomes of these actions shape subsequent stages. This cyclical approach promotes ongoing refinement and alignment of the system with both educational objectives and theoretical insights.

Table 1
Iterative Design Cycle for AI Making in Mathematics Education

Rule-Based System Making Stages	Task	Participatory Action	Consequences
Knowledge Base	Teachers identify gaps in the AI and Mathematics Education literature that can be addressed by making a rule-based expert system. To facilitate this, they design tasks that focus on the performative function of the system.	Students engage with the task by using words, diagrams, AI concepts, and mathematical processes to create a detailed description of how the system performs its functions. They focus on specifying relations, strategies, and heuristics to form the first part of the knowledge base.	Students describe the benefits of this stage for their learning goals and future careers. Teachers analyze students' descriptions to inform subsequent task designs and implications for Mathematics and AI Education.
Explanation Facilities	Based on students' knowledge base descriptions, teachers engage students to explain the AI and mathematical concepts that underpin the system's functions.	Students provide detailed explanations of the AI and mathematical concepts. They articulate how these concepts support the system's performative functions, building on their initial descriptions from the Knowledge Base.	Students reflect on how explaining these concepts enhances their understanding. Teachers use students' explanations to inform further task designs and implications for Mathematics and AI Education.
IF THEN, FACTS, and INFERENCES	Teachers engage students to generate IF-THEN rules, facts, and inferences based on their explanations and knowledge base work.	Students develop and refine IF-THEN rules, facts, and inferences, ensuring alignment with the descriptions and explanations provided in the previous stages.	Students describe the impact of creating these rules on their learning. Teachers analyze students' tasks to inform subsequent task designs and implications for Mathematics and AI Education.
Encoding and Development	Teachers consult with knowledge engineers to develop tasks and select programming software or shells based on students' IF-THEN rules, facts, and inferences.	Students engage with the task to encode their IF-THEN rules, facts, and inferences using the chosen software or shell, applying their reasoning to system development.	Students and teachers assess the alignment between the system's encoded rules and initial descriptions, leading to refinements. Teachers use these insights to inform subsequent task designs and implications for Mathematics and AI Education.
System Testing and Deployment	Teachers and knowledge engineers develop tasks for system testing, revision, and deployment based on the encoded rules and inferences.	Students engage with the task to test, revise, and deploy the rule-based expert system in a real-world educational context, ensuring its functionality aligns with the intended design.	After deployment, students and teachers reflect on the process, describing the benefits and challenges encountered. Teachers analyze these reflections to inform subsequent task designs and implications for Mathematics and AI Education.

As shown in Table 1, in the first stage of the iterative design process for making AI systems in mathematics education, teachers identify gaps in the literature that can be addressed by developing a rule-based expert system. They design tasks focusing on the system's performative function. In the participatory action stage, students use mathematical concepts, AI principles, and diagrams to describe the system's function. This description includes specifying relations, recommendations, directives, strategies, or heuristics for the knowledge base and developing an explanation facility for the underlying AI and mathematical concepts. Subsequently, students and teachers evaluate the consequences of their participation. Students reflect on how their involvement benefits their learning and future careers, while teachers analyze these reflections to inform the design of subsequent tasks. In the next stage, students create IF-THEN rules, facts, and inferences based on their previous work. Teachers consult with knowledge engineers to select suitable programming tools and develop tasks that align with these IF-THEN rules, facts, and inferences. Students then encode their rules and inferences using the chosen tools. The final stage involves testing, revising, and deploying the system. Both students and teachers revisit the consequences of their participation after deployment to assess the process and its educational outcomes.

Methods

Research Design, Study Context and Participants

Given the novelty of the conceptualization and the absence of prior research on its operationalization, the study employs a revelatory case study design (Yin, 2014). This approach offers the first detailed empirical account of the initial iterations of the conceptualized iterative design.

This study is part of an ongoing research project conducted in a high school mathematics classroom setting with 60 students and their teacher in Ghana, West Africa. The aim is to develop AI and mathematics curriculum and tools both authentically and computationally. Currently, this study has engaged 20 of the students to complete the first iteration stage of the design cycle. For convenience, the findings in this study are reported based on analyzed data from a single participant Prince (pseudonym) who has completed the first design iterations.

Data Collection

In the first iteration, I collected data through emails and Google Drive correspondence with the participating students. Over the course of one month, I administered two questionnaires. The first questionnaire included the task, demographic questions, and questions to assess the participants' exposure to AI and their participatory actions. I sent this first questionnaire via email, asking participants to complete the task using general word descriptions, mathematical explanations, and diagrams. Once they completed the task, they emailed it back to me for analysis. In my second email, I instructed the participants to read a half-page paper on AI concepts (Wang & Johnson, 2023). They were then asked to deductively analyze their previously completed tasks based on this reading. Since many participants lacked an AI background and the computational form of the tool was still in development, I gave them an additional task. They had to use characteristics of existing rule-based systems from their households, such as washing machines or robotic cleaners, to show how their analyzed content reflected a feature of these tools. I requested they present their responses in a table with three columns: Envisioned Blueprints for the Yet-to-be-Designed Final Product, AI Concepts and Definition, and Connection in Envisioned Design and Existing Household Rule-Based System. This data would contribute to the explanation facility of the design. After participants completed the second email tasks, I sent them a third email with the second questionnaire, called the Gains questionnaire. This focused on the consequence stage, prompting participants to reflect on their experiences after participating in the study. Once they completed and emailed back this second questionnaire, the data collection stage for the first iteration was concluded.

Task Description

On the initial questionnaire, the description of the task was provided as follows:

> You are tasked with providing a detailed description to be simulated by a rule-based system. At the user interface, the system accepts a specific time input, measured in hours, and generates two distinct outputs. The first output associates the input hour within a 12-hour timeframe with a corresponding description of a geometric transformation. At specific hours, the geometric transformation includes a description of the dragging of vertices of a geometric rectangle, transitioning it into a square. At later hours, the transformation involves further dragging of the square's vertices, resulting in its transformation into a circle. While the system incorporates other geometric transformations, the primary focus is on these three: rectangle to square to circle. The second output is an explanation of the transformation processes, supplemented by images that emphasize the underlying principles of AI, geometry modeling, and constraint satisfaction. The functionality of this rule-based system is entirely based on the simulation of the reasoning processes of expert Culturally Relevant-Sustaining (CRS) mathematics educators. The system is explicitly designed to replicate their reasoning processes, which merge AI methodologies, mathematical concepts, equations, diagrams, and the gaming principles of the Ghanaian three-in-a-row board game, *Kwasiadafranka*. Through this integrated reasoning, the educators describe how the vertices of a rectangle are systematically dragged and repositioned, leading to its transformation into a square and ultimately a circle at specific hourly intervals within the 12-hour framework. Initially, the educators' reasoning processes relied on the use of plain language, mathematical modeling, procedural explanations, and cultural insights derived from

> Kwasiadafranka to illustrate how geometric transformations occur over time. These processes were subsequently encoded into logical structures and programming languages, enabling the system to precisely emulate the reasoning and methodologies employed by the expert educators. As an identified member of this esteemed group of expert CRS mathematics educators, you are tasked with describing your initial reasoning processes. Employing tools such as pen, plain and graph paper, and elements drawn from the three-in-a-row game, you will provide a foundational description of how the vertices of the rectangle are dragged to transition it into a square and subsequently how the square is further transformed into a circle. Your detailed contribution will play a critical role in enhancing the system's ability to simulate reasoning processes while ensuring pedagogical and cultural relevance in CRS mathematics education. Please provide your reasoning below.

Data Analysis

To describe the participants' actions, I employ deductive and inductive analysis (Bingham, 2023). I deductively analyzed the contents of the first email, which included their responses to the task questionnaire. I used tables with three headings: gamification, general word description, mathematical description, and use of diagrams. I quoted statements directly from the questionnaire under each heading and added interpretations where necessary. Next, I analyzed the participants' second email, which contained their deductive analysis of AI concepts. I focused on whether their analysis aligned with their initial task descriptions. I mainly read the responses to check the correctness of their content. Finally, I used the prompts from the Gains questionnaire to deductively analyze the participants' third email, where they reflected on the consequences of their participation. I then read all the analysis inductively to describe the designs impact on participants, as well as implications for practice and task design for the next cycle.

Results

As shown in Table 2 and Figures 2-4, to describe the transformation from a rectangle to a square, Prince employed the gaming principle and features of (a) positioning and repositioning pebbles on the vertices along the lengths of a rectangular board game (often drawn on paper or floors but in his case, graph sheets), and (b) using rounded papers instead of pebbles to depict the nine vertices of the rectangular board game. As presented in Table 2, Prince transformed the rectangle into a square by repositioning all the rounded papers on the three vertices of each of the two widths of the rectangular board to new positions.

Since he initially positioned the rounded papers to get the rectangle (Figure 2), he knew the initial rectangle had lengths of 24 cm and a width of 20 cm. Hence, to convert it to a square, he needed to make the lengths shorter from 24cm to 20 cm. To achieve this (Figures 3 and 4), he moved all the rounded papers at the widths of the rectangle toward the vertical symmetry axis. He relocated each of them 2 cm from their original positions toward the vertical symmetry axis, resulting in a total reduction of 4 cm from the initial width of 24 cm to a new width of 20 cm, matching the length.

Table 2

Analysis of Prince's Participatory Actions: Rearranging a Rectangle, to a Square

Process	Gamification (direct quote)	Word Use (direct quote)	Math Processes (interpreted)	AI Concept (direct quote)
Rectangle to Square	I applied the game concept of moving a pebble by dragging it horizontally, determining both the direction and distance relative to a reference point. To ensure precise measurements, I used graph sheets and cut out circular shapes from brown paper as pebbles. Each cut-out piece was carefully positioned on the graph, with coordinates assigned to them, and I printed a name on the back of each round paper for identification.	I vertically aligned three rounded papers defining the rectangle's width to new coordinates. Knowing the rectangle's dimensions (24 cm length, 20 cm width), I aimed to make the lengths equal for a square. Utilizing taxicab geometry, I moved all rounded papers at the rectangle's widths 2 cm towards the vertical symmetry axis. This adjustment ensured equal sides for the resulting square. I verified the accuracy by calculating absolute differences in old and new x and y coordinates.	- Identifying the rectangle's dimensions: length = 24 cm, width = 20 cm - Converting the rectangle into a square by adjusting the lengths to 20 cm - Utilizing taxicab geometry to move papers 2 cm towards the vertical symmetry axis - Verifying accuracy through absolute differences in x and y coordinates to maintain symmetry	Classical Search: Taxicab geometry for horizontal distance calculation mirrors Manhattan heuristic in A* search algorithms

Figure 2
Initial Display as a Rectangle

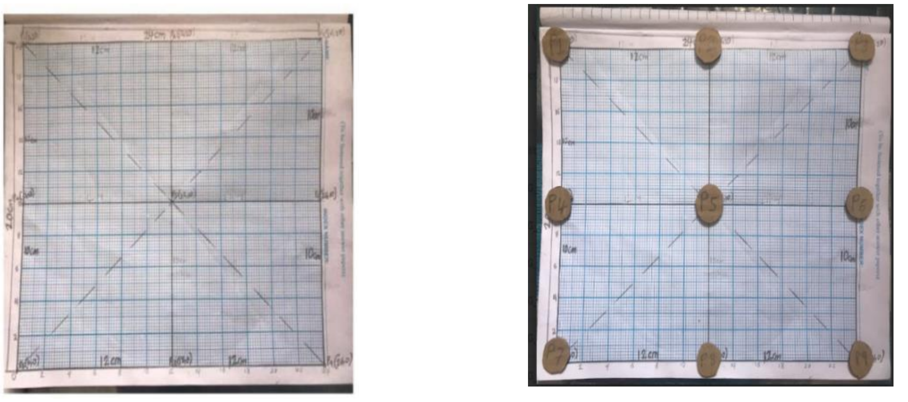


Figure 3
Rearranging from Rectangle to Square

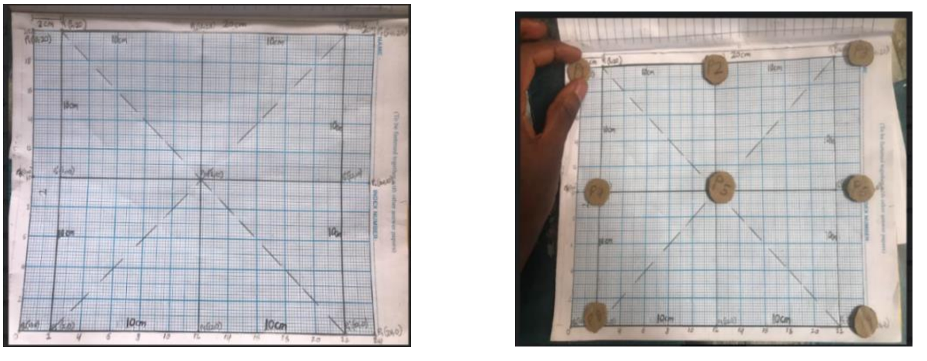
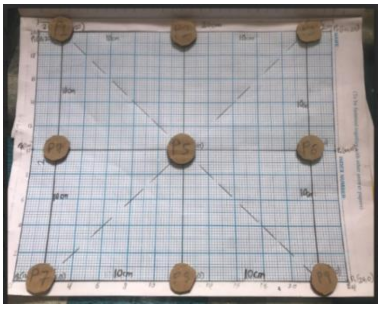


Figure 4
Rearranged Rectangle to Square



As shown in Figure 5 below, he verified if the' distance between the new and old locations was 2 cm by calculating the absolute differences in the respective old and new x and y coordinates, stating that he applied his idea of taxicab geometry. As shown in Table 2, Prince stated the AI concept of classical search centrally mirrors his design. As shown in Figure 2 above, this is seen in his use of explicit measurements, as seen in the graphs and in the use of taxicab geometry. Prince stated that his approach to applying taxicab geometry and horizontal direction of moving the round papers mirrors the processes of the Manhattan heuristic in A* search algorithms. Hence, he envisages that the product will search for the new location of the points to be dragged using the A* search algorithm.

Figure 5
Displayed Mathematical Calculations

Applying Taxicab geometry to calculate the distance between the initial and target position

***P*1**

- Initial: (0,20)
- Target: (2,20)
- Distance: |0−2|+|20−20|=2+0=2

***P*4**

- Initial: (0,10)
- Target: (2,10)
- Distance: |0−2|+|10−10|=2+0=2

***P*3**

- Initial: (24,20)
- Target: (22,20)
- Distance: |24−22|+|20−20|=2+0=2

***P*6**

- Initial: (24,10)
- Target: (22,10)
- Distance: |24−22|+|10−10|=2+0=2

***P*7**

- Initial: (0,0)
- Target: (2,0)
- Distance: |0−2|+|0−0|=2+0=2

***P*9**

- Initial: (20,0)
- Target; (22,0)
- Distance: |24-22| + |0-0| = 2 + 0 = 2

Rearranging from Square to Circle

As shown in Table 3 and Figures 6-8, to describe the transformation from a square to a circle, Prince employed the gaming principle and features of positioning and repositioning pebbles on the vertices along the diagonal of the rectangular board game. As shown in Table 3 below, Prince noted that rearranging the square into a circle was more challenging. Initially, he tried dragging the rounded papers horizontally or vertically using taxicab geometry, but this did not work. He then considered the Euclidean distance formula and diagonal dragging. Prince realized that the center of the square was equidistant from its symmetric vertices but not from its diagonal vertices. He then moved the diagonal vertices along the diagonal lines toward the center, making their distances equal to the equidistance between the symmetric vertices and the center. By adjusting the vertices to these calculated distances, he successfully transformed the square into a circle (Figures 6 & 7). As shown in Figure 8, to confirm the attainment of the equal lengths, he calculated the length between the new positions of the dragged vertices and that of the center using the Euclidean distance formula. In describing the AI concept he employed at this stage, Prince explained that his approach to moving diagonally and calculating the distance along the diagonal using Euclidean geometry counters the Manhattan heuristic processes. He further read outside the analytical lens and found that this approach to calculating distances along diagonal lines using the Euclidean distance formula is a heuristic used in the Greedy Best-First Search algorithm. In this instance, he stated that the product would be using this search approach to locate the diagonal points.

Table 3
Analysis of Prince's Participatory Actions: Rearranging a Square to Circle

Process	Gamification (direct quote)	Word Use (direct quote)	Math Processes (interpreted)	AI Concept
Square to circle	Guiding Game Principle: I applied the game concept of moving a pebble by dragging it diagonally, determining both the direction and distance relative to a reference point. To ensure precise measurements, I used graph sheets and cut out circular shapes from brown paper as pebbles. Each cut-out piece was carefully positioned on the graph, with coordinates assigned to them, and I printed a name on the back of each round paper for identification.	Initially, I attempted horizontal or vertical movements using taxicab geometry, but they were unsuccessful. Realizing the center's equidistance to symmetric vertices but not to diagonal vertices, I dragged the diagonal vertices towards the center. I aimed to equalize their distance to that between symmetric vertices and the center. By calculating distances using the Euclidean distance formula, I adjusted the vertices, successfully transforming the square into a circle.	- Recognizing the square's center's equidistance to symmetric vertices but not to diagonal vertices - Dragging diagonal vertices towards the center to equalize distances - Calculating distances using the Euclidean distance formula - Adjusting vertices to calculated distances to transform the square into a circle	Classical Search: Euclidean geometry for diagonal distance calculation mirrors Euclidean distance heuristic used in the Greedy Best-First Search algorithm

Figure 6
Initial Display as a Square

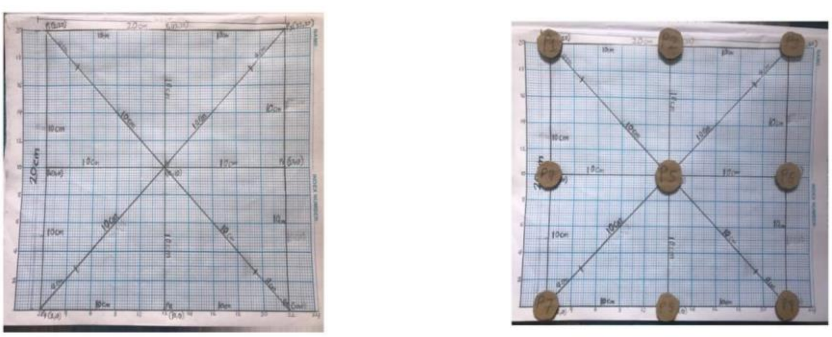
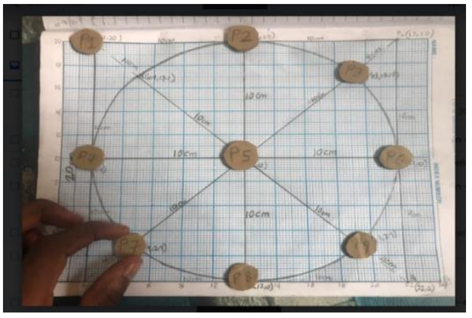


Figure 7 (A & B)

A:Rearranging from Square to Circle *B: Rearranged Square to Circle*



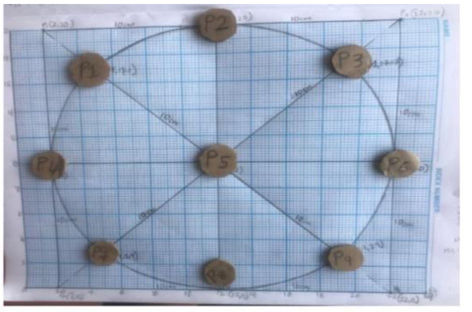
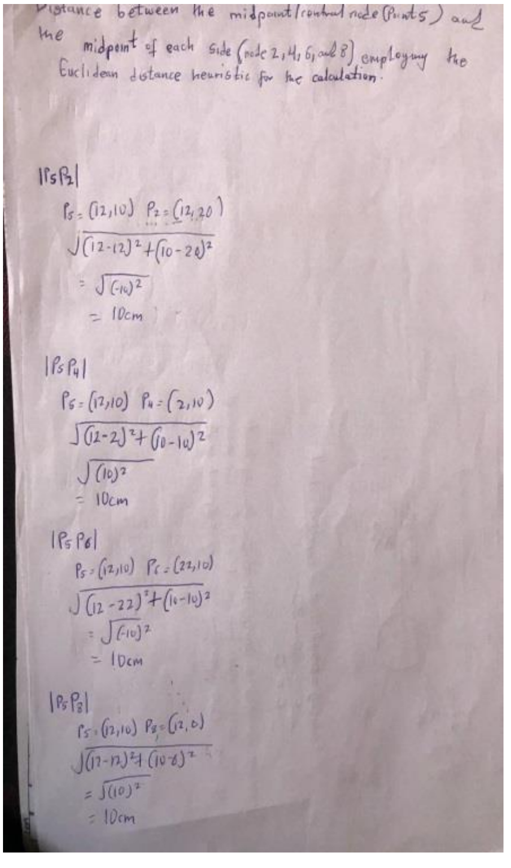


Figure 8

Displayed Mathematical Calculations



Distance between the midpoint/central node (Point 5) and the midpoint of each side (node 2, 4, 6, and 8) employing the Euclidean distance heuristic for the calculation.

$|P_5P_2|$

$P_5 = (12,10)$ $P_2 = (12,20)$

$\sqrt{(12-12)^2+(10-20)^2}$

$= \sqrt{(-10)^2}$

$= 10cm$

$|P_5P_4|$

$P_5 = (12,10)$ $P_4 = (2,10)$

$\sqrt{(12-2)^2+(10-10)^2}$

$\sqrt{(10)^2}$

$= 10cm$

$|P_5P_6|$

$P_5 = (12,10)$ $P_6 = (22,10)$

$\sqrt{(12-22)^2+(10-10)^2}$

$= \sqrt{(-10)^2}$

$= 10cm$

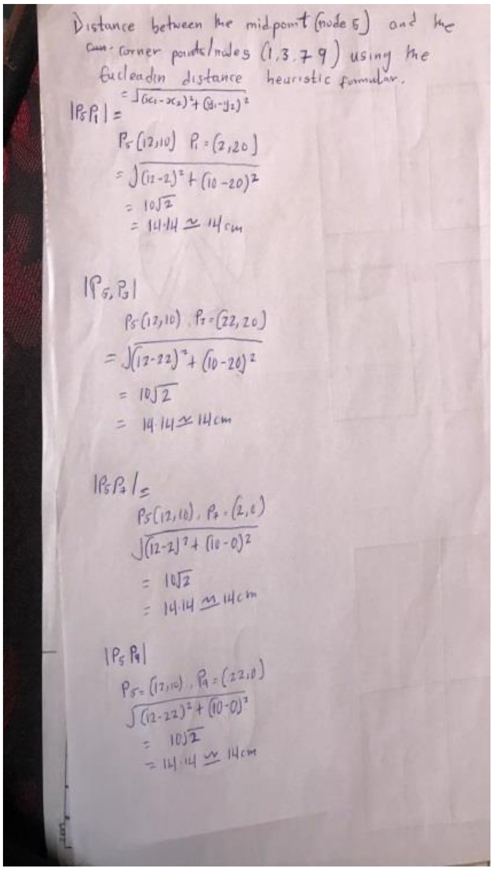
$|P_5P_8|$

$P_5 = (12,10)$ $P_8 = (12,0)$

$\sqrt{(12-12)^2+(10-0)^2}$

$= \sqrt{(10)^2}$

$= 10cm$



Distance between the midpoint (node 5) and the corner points/nodes (1, 3, 7, 9) using the Eucleadin distance heuristic formular.

$|P_5P_1| = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$P_5 (12,10)$ $P_1 = (2,20)$

$= \sqrt{(12-2)^2+(10-20)^2}$

$= 10\sqrt{2}$

$= 14.14 \simeq 14cm$

$|P_5, P_3|$

$P_5 (12,10)$, $P_7 = (22,20)$

$= \sqrt{(12-22)^2+(10-20)^2}$

$= 10\sqrt{2}$

$= 14.14 \simeq 14cm$

$|P_5P_7| =$

$P_5 (12,10)$, $P_7 = (2,0)$

$\sqrt{(12-2)^2+(10-0)^2}$

$= 10\sqrt{2}$

$= 14.14 \simeq 14cm$

$|P_5P_9|$

$P_5 = (12,10)$, $P_9 = (22,0)$

$\sqrt{(12-22)^2+(10-0)^2}$

$= 10\sqrt{2}$

$= 14.14 \simeq 14cm$

Connection Between Design Features of the System and AI Systems in Prince's Household

As shown in Table 4, Prince described the decision-making process of the envisaged system as following the constraint satisfaction problem paradigm, where each transition is required to meet predefined criteria. He compared this to the operation of his household washing machine, which similarly operates within a set of constraints, such as adjusting its cycle based on the water level, ensuring that specific conditions are satisfied before moving to the next stage. This analogy highlights the shared principle of satisfying conditions to progress through stages in both systems.

Table 4

Prince's Participatory Actions: Analysis of AI Concepts Based on the Task

AI Concepts (Direct Quote)	Connection in Envisioned System Design and Existing Household Rule-Based System (Direct Quote)
Pathfinding and Motion Planning: involve algorithms for moving from one point to another while avoiding obstacles, using heuristics and optimization techniques.	**Envisioned Design:** Encoding the systems knowledge base involves planning the movement of points as it transforms shapes, like navigating through a series of transitions. **Robot Vacuum Cleaner:** Uses pathfinding to navigate around obstacles and motion planning to adjust its cleaning path efficiently.
Rule-Based Decision Making: uses predefined rules to make decisions or take actions based on specific conditions, following if-then logic.	**Envisioned Design:** Managed by predefined rules dictating how and when points are moved, and shapes are changed. **Coffee Maker with Timer:** Uses rules based on time and water level to control brewing, like the systems rules for transformation.
Constraint Satisfaction Problem: Involves finding solutions that satisfy a set of constraints or conditions, optimizing arrangements or values to meet criteria.	**Envisioned Design:** Satisfies constraints related to point positioning and shape dimensions during transformations. **Washing Machine Cycle:** Operates based on constraints such as water levels and cycle completions. **Smart Lighting System:** Makes decisions based on constraints like time and ambient light levels.
Geometric Transformations: Adjusts shapes or objects through operations like translation, rotation, scaling, or reflection to manipulate geometric entities.	**Envisioned Design:** Involves geometric transformations to change the frame's shape from one geometric form to another. **Robot Vacuum Cleaner:** Adjusts its path and direction based on obstacles, involving geometric transformations in navigation and motion planning.

Princes Envisaged Description of the Mechanisms of the Expert System

As quoted below, to describe the envisaged description of the mechanism of the system, Prince stated that the system would have its knowledge base as his descriptions of how a geometric rectangle, transition to the next shapes, circle and square after every four hours. Prince stated that the first four hours (12-3) of a clockwise direction, embodies his descriptions of how the vertices of the rectangle transition to a square. Similarly, he stated that the next four hours (4-7), embodies his descriptions of how a square transitions to a circle. In the remaining, five hours (8-12), Prince stated that it reflects the descriptions of the cycling back from a square, through a circle to the initial design state, a rectangle.

> In terms of the product, I envision a system where transformations align with a clockwise progression of hours, following a logical sequence of transitions. For the first range of hours (12-3), the system describes the transformation from a rectangle to a square. **If** a user inputs 1 or 2 hours, **then** the output will display a description of how three out of six points along the rectangle's width relocate to new positions, forming a square. **If** the user inputs 3 hours, **then** the system will output a complete description of the rectangle-to-square transformation. For the second range of hours (4-7), the focus shifts to the transition from a square to a circle. **If** a user inputs 4 hours, **then** the output will describe how the diagonals of the square are positioned to calculate the radius of the circle. **If** the user inputs 5 hours, **then** the system will output a description of the equidistant relationship between the square's side length and the circle's radius. **If** the user inputs 6 or 7 hours, **then** the system will describe the final stages of the transition from a square to a circle. For the final range of hours (8-12), the system describes the reverse transformations, moving from a circle back to a square and ultimately to a rectangle. **If** a user inputs any hour within this range, **then** the output will display a description of the transition stages, concluding with the rectangle's restoration at 12 hours. These transformations, supported by corresponding images, artificial intelligence functionalities, and mathematical concepts, are visualized within an explainable facility. This design integrates both aesthetic appeal and functional utility, offering users an interactive and engaging experience through its shape-shifting capabilities.
> (Direct quote from Princes envisioned blueprints for the yet-to-be-designed final product, 01/29/24).

Iterative Design Stage 3: Consequences

To illustrate the details of consequences, a quotation from Prince is included (Quoted from Prince's Summary on the Gains Questionnaire, Page 3 on 01/29/24). As Prince described:

> I enjoyed the design process very much. Some aspects really pushed me to think about the structural and dependent relationships between basic 2D geometric shapes that I didn't know before. During the stage where I had to identify which AI concepts my work connects with, I was initially struck by the insight that some trivial, technical mathematical processes meant the same thing for some AI concepts. For example, I was surprised that the processes of dragging based on directions, as used in playing the game, involved a lot of mathematical formulas and calculations such as taxicab and Euclidean geometry and absolute value, which concurrently become heuristics like Manhattan pathfinding used in types of AI local searching. This wasn't just an isolated instance; there were many more that informed me about how our already known mathematical approaches and game-playing concepts can be used as a context for understanding some technical AI concepts. I acknowledge that some stages were too demanding and difficult, especially the transition from a square to a circle. Yet, when I figured it out, I enjoyed the opportunity

> to learn. Through this activity, I have acquired some understanding of how AI works, even though we are yet to move to the computational design aspect. I feel surreal about potentially having a career in AI, even if it might not be my main job. I would rather be a mathematician who applies abstract mathematics to AI designs, sells them, and makes more money and signature developments. Understanding that an AI tool is to behave based on my knowledge is so important for me. I have come to realize the background work of what AI experts go through. I am confident in designing more since it is just an application of some of the mathematics and gaming principles I know. At some point, I didn't realize that I was using mathematics, but when I was writing this reflection, I understood that I applied various geometric transformations and distance measurements. One day, I will just sell algorithms.

As seen in this quote, Prince expressed several significant consequences from participating in the design stages, highlighting that he was pushed to think deeply, leading to new insights about the structure of 2D geometric objects. He became aware of the relevance of his gamified concepts to both mathematics and AI, which has been pivotal in creating mathematical algorithms for AI tools. Prince expressed a sustained interest in future design stages, eager to see how an AI tool will embody his decisions and represent them as knowledge. These reflections highlight how the design process has been instrumental in cultivating Prince's sustained interest in mathematics, AI, and related future career paths.

Discussion

This paper discussed a three-stage iterative participatory design: task design, participatory actions, and consequences, aimed at engaging high school students in making AI tools for mathematics education. The first iteration was operationalized by assigning students the task of using pen and paper to describe, with words, diagrams, mathematical processes, and AI concepts, the predefined rules and constraints for a yet-to-be-developed AI tool. This tool functions based on its simulations of students' reasoning processes regarding: (a) hourly changes in the position of the vertices of a 2D shape to form a new shape, and (b) displayed transformation processes, including geometry modeling and constraint satisfaction. As a rule-based system, when a specific time in hours is entered, the input is processed by the inference engine. This engine aligns with the students' encodings of the rule "IF it is A-hour, THEN the arrangement of the vertices is B."

The current paper examines the description of students' reasoning processes in relation to the function of the system and its explainability. The analysis of one student's description revealed numerous opportunities. In mathematics education, the initial iteration can be tailored as personalized learning scaffolds (Walkington et al., 2024). This approach helps students understand the relevance of mathematics to careers such as AI tool design. In

AI and mathematics education, the initial iteration functions as an unplugged AI curriculum (Nishida et al., 2019) and a contextualized AI literacy process (Chiu et al., 2024). This approach helps students understand AI knowledge representations, planning, and associated mathematical concepts and processes without utilizing AI tools. In terms of the design iterations, the findings show that students can use their descriptions as tasks in the remaining design stages. Students will refine them into logics, inferences, encoding, testing, revision and deployment. Despite the valuable insights gained from the initial iteration, a significant challenge persists in engaging with students individually. Their differing knowledge bases lead to the development of personalized rule-based systems, making it difficult to apply a uniform approach across participants.

In conclusion, the insights revealed from the initial iteration of the design highlight its potential for student involvement in formulating predefined steps essential for making AI tools in mathematics education. These pre-defined steps guide subsequent iterations, showcasing the iterative nature of the design process for making AI tools in mathematics education with high school students.

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